5.1, 5.2, 5.3 – Properites of Exponents

last revised 6/7/2014

Properites of Exponents

1. $x^a \cdot x^b = x^{a+b}$

2. $\frac{x^a}{x^b} = x^{a-b}$

3. $(x^a)^b = x^{a \cdot b}$

4. $x^0 = 1$

5. $x^{-a} = \frac{1}{x^a}$

*Simplify each of the following:

a. $x^4 \cdot x^8 =$

b. $x^5 \cdot x^7 \cdot x =$

c. $5^6 \cdot 5^{11} =$

d. $\frac{x^{14}}{x^9} =$

e. $\frac{x^6 y^{11} z^{14}}{x^3 y^7 z^{12}} =$

f. $(2x^2 y^{15,000})^0 =$

g. $3x^0 =$

h. $(3x)^0 =$

i. $\frac{x^{-4}}{x^2} =$

j. $\frac{x^3 y^{-4}}{x^{-2} y^8} =$

Negative exponents are NOT considered to be simplified. Do NOT leave them in final answers!

k. $\left(\frac{3}{5}\right)^2 =$

l. $(3x^2y^3)^4 =$

m. $(2x^2y^3)^2(-3xy^4)^2 =$

n. $\frac{x^{-4}}{x^{-8}} =$

o. $\frac{5^{-1}}{5} =$

p. $6^{-2} =$

q. $-6^{-2} =$

r. $\frac{-12x^{-4}y^{-3}}{48x^{-7}y^5} =$

s. $\left(\frac{3x^{-2}y^4}{6x^5y^{-7}}\right)^{-3} =$

6. Simplify: $\left(\frac{-3u^{-3}}{w^{-6}}\right)(-2u^2v^3w^2)^{-3}$

Simplify each:

$5x^{-3}$ $7^3 \cdot 7^{11}$ $5(x^2y^3)^0$

5.4 – Scientific Notation

Scientific notationis a shorthand notation for writing extremely small or large numbers.

Notation:

*Write each using scientific notation:

1. 9,374,000

2. 19.4 trillion

3. 0.000381

*Write each in standard form:

4. $4.71 \text{ x } 10^{8}$

5. $3.21 \text{ x } 10^{-5}$

*Multiply. Write your answers in scientific notation:

6. $\left(3.5 \text{ x } 10^{11}\right)\left(4.0 \text{ x } 10^{23}\right)$

7. $\left(2.45 \text{ x } 10^{17}\right)\left(3.5 \text{ x } 10^{12}\right)$

*Divide. Write your answers in scientific notation:

8. $\frac{12.5 \text{ x } 10^{-4}}{2.5 \text{ x } 10^{19}}$

9. $\frac{2.4 \text{ x } 10^{8}}{4.8 \text{ x } 10^{42}}$

5.5 – Adding and Subtracting Polynomials

monomial

binomial

trinomial

polynomial

Vocabulary: $ax^n + bx^{n-1} + \cdots + cx + d$

*Given: $5x^7 + 4x^6 + 3x^5 \cdots + 5x - 11$, find the following:

a. leading coefficient

b. constant term

c. degree of the second term

d. degree of the polynomial

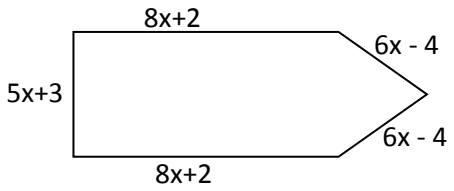
If a term has more than one variable, its degree is the _____ of its exponents.

*What is the degree of the expression $5x^2y^7$?

*Add: $(3x^2 + 5x - 2) + (7x^2 - 9x + 13)$

*Add:
$$\begin{array}{r} 5x^3+3x^2 \quad +11 \\ \underline{8x^3-9x^2+5x-3} \end{array}$$

*Find the perimeter:



*Subtract: $(3x^2 + 5x + 11) - (x^2 + 7x - 4)$

*Subtract:
$$\begin{array}{r} 5x^3-2x^2+4x-9 \\ \underline{2x^3+7x^2-11x+8} \end{array}$$

5.6 – Multiplying Polynomials

*Multiply each of the following:

1. $(-3x^4)(5x^5)$

2. $4x(3x-7)$

3. $5a^2b^3c^4(3ab^7-5ab^2d^5)$

4. $(x+5)(x-3)$

5. $(2x-3)(3x+5)$

6. $(3x-5)(2x+4)$

7. $(5x-1)(x+8)$

8. $(4x-7)(4x+7)$

9. $(a+b)(c+d)$

10. $(3x-2)(x^2+4x-7)$

11. $(5x+7)(3x-2)$

12. $(3x-2)^2$

13. $(3x^2y^4)^2$

Mini-Review of Sections 5.1 – 5.6

1. Simplify: $\frac{-5x^{-3}y}{10x^4y^{-5}}$

2. Multiply: $\left(4.3 \text{ x } 10^{8}\right)\left(3.0 \text{ x } 10^{17}\right)$

3. Divide: $\frac{0.6 \text{ x } 10^{-15}}{2.4 \text{ x } 10^{11}}$

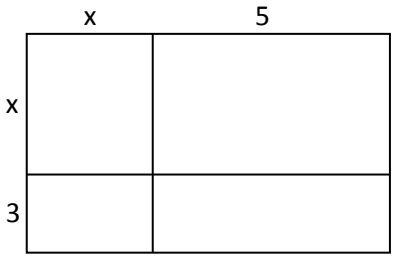
4. Multiply: $-5x^2y^3z(8xyz^5 - 4xy^2z^3)$

5. Multiply: $(3x^2 + 4)(2x^2 - 7)$

6. Multiply: $(x - 3)(x^2 + 3x + 9)$

7. Multiply: $(x^2 + 4x - 2)(x^2 + x - 5)$

8. Calculate the area of the larger rectangle in two ways.



5.7 – Dividing Polynomials

A. Dividing by a monomial

Create separate fractions and then simplify each separately.

1. $\frac{10x^4y^3+15x^2y^8}{10x^2y^9}$

2. $(8x^3y - 4x^7y^5 + 2x^2y^4) \div (4xy^8)$

B. Dividing by a non-Monomial

Use long division.

Recall... $512 \div 31$

3. $(x^3 + 4x^2 - 2x + 8) \div (x - 1)$

4. $\frac{4t^3+4t^2-9t+3}{2t+3}$

5. $(4x^3 + 5x - 12) \div (2x - 3)$

6. $(x^3 - 27) \div (x - 3)$

7. $(p^4 - p^3 - 4p^2 - 2p - 15) \div (p^2 + 2)$

In Math 90, you will learn another method for dividing called Synthetic Division. This process will only work when dividing by a linear factor (those without exponents). Problems like #7 above cannot be done using synthetic division since there is an exponent in the divisor.

6.1 – Introduction to Factoring

Factoring is _____ .

Factoring is a _____ process.

A. Factoring Out a Greatest Common Factor

*Factor each of the following completely.

1. $24x - 36$

2. $18x^2 - 18x$

3. $20x^5y^3z^2 - 24x^2y^5z$

4. $14x^5y^3 - 28x^7y^2 + 35x^2y^8$

5. $-12x^3 + 4x^2 - 9$

B. Factoring by Grouping

6. $x^2(x-5) + 7(x-5)$

7. $5x(x^3+2) - 8(x^3+2)$

8. $3q + 3p + qr + pr$

9. $8w^5 + 12w^2 - 10w^3 - 15$

10. $2c + 3ay + ac + 6y$

11. $12x^2 + 6x + 8x + 4$

12. $6f^2k + 30k + 2f^2 + 10$

Review

1. Multiply: $(x + 1)(x + 2)(x + 3)$

2. Simplify: $\frac{5^3}{5^{-8}}$

3. Simplify: $(5x^3y^2)^{-2}$

4. Simplify: $\left(\frac{4x^{-3}y}{6xy^{-5}}\right)^{-3}$

5. Multiply: $\left(5.6 \text{ x } 10^{14}\right)\left(3.0 \text{ x } 10^{22}\right)$

6. Multiply: $-4x^2y^3z(8xy^5 - 11x^2z^3)$

7. Multiply: $(5x^2y - 8z)^2$

8. Divide:
$(16x^4y^2 + 20x^5y - 24x^3y^8) \div (6x^2y^7)$

9. Divide: $(x^3 + 6x - 7) \div (x + 1)$

10. Divide: $(m^3 - 64) \div (m - 4)$

11. Factor: $-8x^3 + 16x^2 + 20x$

12. Factor: $5x^2(x + 7) - 8(x + 7)$

13. Factor: $2c + ac + 3ay + 6y$

14. Factor: $xy - xz + 7y - 7z$

15. Factor: $4x^3 + 3x^2y + 4xy^2 + 3y^3$

6.2 – Factoring Trinomials, part 1

* Factor each of the following:

1. $x^2 + 10x + 16$

2. $x^2 - 3x - 18$

3. $x^2 + 6x - 40$

4. $m^2 - 12m + 11$

5. $n^2 + 8n + 16$

6. $w^2 - 7w + 12$

7. $12p^2 - 96p + 84$

8. $x^3y^3 - 19x^2y^3 + 60xy^3$

9. $-2m^2 + 22m - 20$

10. $5w^2 - 40w - 45$

6.3 – Factoring Trinomials, part 2

pattern: $ax^2 + bx + c$

*Factor each of the following completely.

1. $3x^2 + 13x + 4$

2. $7y^2 + 9y - 10$

3. $8 + 7x^2 - 18x$

4. $12c^2 - 5c - 2$

5. $12y^2 - 73yz + 6z^2$

6. $36x^2 - 18x - 4$

7. $12m^2 + 11mn - 5n^2$

8. $y^2 - 6y - 40$

9. $16x^2 + 24x + 9$

10. $6p^4 + 17p^2 + 10$

11. $3y^3 - y^2 + 12y$

6.4 – Factoring Trinomials, part 3

The a-c Method

1. $6x^2 + 7x - 3$

2. $9x^2 - 12x + 4$

3. $16x^2 + 10x + 1$

4. $16x^2 - 34x - 15$

6.5 – The Difference of Two Squares and Perfect Square Trinomials

The Difference of Two Squares

*Factor each completely:

1. $x^2 - 49$

2. $x^2 - 64$

3. $x^2 - 25$

4. $x^2 - 10$

5. $x^2 - \frac{1}{36}$

6. $x^2 + 25$

7. $x^2y^2 - 100z^2$

8. $x^4 - 16$

9. $x^8 - y^8$

10. $x^2 - 1$

11. $25x^2 - 16$

12. $100x^2 - 49y^2$

13. $25x^2 - 100$

14. $x^2 - 6xy + 9y^2 - 16$

15. $x^2 + 8x + 16 - 25y^2$

16. $w^2 - 10w + 25 - 36q^2$

17. $100 - x^2 - 16xy - 64y^2$

6.6 – The Sum & Difference of Two Cubes

Memorize:

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$
$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

"SOAP" means....

*Factor each completely:

1. $x^3 + 125$

2. $y^3 - 64$

3. $8x^3 - 1$

4. $3x^3 + 81$

What if you forget these formulas?

A General Stategy for Factoring

1. Can I factor out a _____ ?
2. How many terms are there?
   a. if four, try _____.
   b. If three, try _____.
   c. If two, try _____
      or try _____.
3. Can I factor further?

The following exercises are from pages 470-471 of your textbook, which are printed on the next page for your use.

#5. $2a^2 - 162$

#15. $3q^2 - 9q - 12$

#25. $64 + 16k + k^2$

#10. $5r^3 + 5$

#30. $3y^2 + y + 1$

#35. $-p^3 - 5p^2 - 4p$

#45. $14u^2 - 11uv + 2v^2$

#55. $81u^2 - 90uv + 25v^2$

#65. $12x^2 - 12x + 3$

5. $2a^2 - 162$

6. $y^2 + 4y + 3$

7. $6w^2 - 6w$

8. $16z^4 - 81$

9. $3t^2 + 13t + 4$

10. $5r^3 + 5$

11. $3ac + ad - 3bc - bd$

12. $x^3 - 125$

13. $y^3 + 8$

14. $7p^2 - 29p + 4$

15. $3q^2 - 9q - 12$

16. $-2x^2 + 8x - 8$

17. $18a^2 + 12a$

18. $54 - 2y^3$

19. $4t^2 - 100$

20. $4t^2 - 31t - 8$

21. $10c^2 + 10c + 10$

22. $2xw - 10x + 3yw - 15y$

23. $x^3 + 0.001$

24. $4q^2 - 9$

25. $64 + 16k + k^2$

26. $s^2t + 5t + 6s^2 + 30$

27. $2x^2 + 2x - xy - y$

28. $w^3 + y^3$

29. $a^3 - c^3$

30. $3y^2 + y + 1$

31. $c^2 + 8c + 9$

32. $a^2 + 2a + 1$

33. $b^2 + 10b + 25$

34. $-t^2 - 4t + 32$

35. $-p^3 - 5p^2 - 4p$

36. $x^2y^2 - 49$

37. $6x^2 - 21x - 45$

38. $20y^2 - 14y + 2$

39. $5a^2bc^3 - 7abc^2$

40. $8a^2 - 50$

41. $t^2 + 2t - 63$

42. $b^2 + 2b - 80$

43. $ab + ay - b^2 - by$

44. $6x^3y^4 + 3x^2y^5$

45. $14u^2 - 11uv + 2v^2$

46. $9p^2 - 36pq + 4q^2$

47. $4q^2 - 8q - 6$

48. $9w^2 + 3w - 15$

49. $9m^2 + 16n^2$

50. $5b^2 - 30b + 45$

51. $6r^2 + 11r + 3$

52. $4s^2 + 4s - 15$

53. $16a^4 - 1$

54. $p^3 + p^2c - 9p - 9c$

55. $81u^2 - 90uv + 25v^2$

56. $4x^2 + 16$

57. $x^2 - 5x - 6$

58. $q^2 + q - 7$

59. $2ax - 6ay + 4bx - 12by$

60. $8m^3 - 10m^2 - 3m$

61. $21x^4y + 41x^3y + 10x^2y$

62. $2m^4 - 128$

63. $8uv - 6u + 12v - 9$

64. $4t^2 - 20t + st - 5s$

65. $12x^2 - 12x + 3$

66. $p^2 + 2pq + q^2$

67. $6n^3 + 5n^2 - 4n$

68. $4k^3 + 4k^2 - 3k$

69. $64 - y^2$

70. $36b - b^3$

71. $b^2 - 4b + 10$

72. $y^2 + 6y + 8$

73. $c^4 - 12c^2 + 20$

6.7 – Solving Equations Using the Zero Product Rule

Quadratic equations are of the form

Zero Product Rule:
If $A \cdot B = 0$, Then $A = 0 \ \ or \ B = 0$.

Solve each of the following equations.

1. $(x + 2)(x - 3) = 0$

2. $(x + 5)(2x - 3) = 0$

3. $(8x - 5)(7x + 2) = 0$

4. $\frac{1}{3}z\left(z - \frac{5}{8}\right) = 0$

5. $x^2 - 2x - 15 = 0$

6. $x^2 - 8x + 16 = 0$

7. $x^2 - 24 = 2x$

8. $x^2 - 25 = 0$

9. $2x^2 - 50 = 0$

10. $x^3 - 25x = 0$

11. $2x^3 - 50x = 0$

6.7 #26 $y^2 - 7y - 8 = 0$

6.7 #28 $w^2 - 10w + 16 = 0$

6.7 #30 $4x^2 - 11x = 3$

6.7 #35 $2m^3 - 5m^2 - 12m = 0$

6.7 #38 $4(2x - 1)(x - 10)(x + 7) = 0$

6.7 #46 $2y^2 - 20y = 0$

6.7 #48 $9n^2 = 1$

6.7 #49 $2y^3 + 14y^2 = -20y$

6.7 #62 $3z(z - 2) - z = 3z^2 + 4$

6.7 #69 $(x - 1)(x + 2) = 18$

6.8 – Quadratic Word Problems

A. Number Problems

6.8 #10 If a number is added to two times its square, the result is 36. Find all such numbers.

6.8 #16 The sum of the squares of two consecutive integers is 9 less than 10 times their sum. Find all such integers.

B. Consecutive Integer Problems

Review...

Consecutive Integers

Consecutive Even Integers

Consecutive Odd Integers

6.8 #14 The product of two even consecutive integers is 48. Find all such numbers.

The product of two consecutive odd integers is 63. Find all such integers.

The perimeter of a rectangle is 22 cm and its area is 24cm^2. Find the length and width of this rectangle.

C. Area Problems

6.8 #18 The width of a rectangular picture frame is 2 inches less than its length. The area is 120 in^2. Find its dimensions.

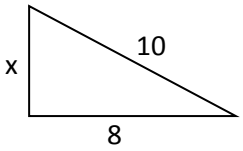
The length of a rectangle is three times its width. Find the dimensions if the area is 48 cm^2.

D. Height of a Projectile

6.8 #26 A stone is dropped off a 256-ft. cliff. The height of the stone is given by $h = -16t^2 + 256$, where *t* is the time (in seconds). When will it hit the ground?

E. The Pythagorean Theorem

Find x:



6.8 #42 The longer leg of a right triangle is 1 cm less than twice the shorter leg. The hypotenuse is 1 cm more than twice the shorter leg. Find the length of the shorter leg.

Review of Chapters 5 & 6

1. Simplify: $(5x^2y)(-3x^4y^3)$

2. Simplify: $5x^0$

3. Simplify: 3^{-3}

4. Simplify: $(x^5)^3$

5. Simlify: $\left(\frac{3x^{-2}y^4}{6x^5y^{-2}}\right)^{-3}$

6. Simplify: $\left(\frac{-4x^{-5}y^{-3}z}{8x^{-7}y^4z^{-3}}\right)^{-4}$

7. Write $5.37 \text{ x } 10^4$ in standard form.

8. Write 365,000,000 in scientific notation.

9. Multiply: $\left(3.5 \text{ x } 10^{11}\right)\left(4.0 \text{ x } 10^{23}\right)$

10. Divide: $\frac{6.0 \text{ x } 10^4}{8.0 \text{ x } 10^{23}}$

11. Divide: $\frac{x^3+64}{x+4}$

12. Divide:

$(5x^3 + 10x^2 - 15x + 20) \div (15x^3)$

13. Simplify: $(3x - 2y)^2$

14. Add: $(4x + 2) + (3x - 1)$

15. Subtract $3x^2 - 4x + 8$ from $x^2 - 9x - 11$.

16. Multiply: $(3x + 5)(2x - 7)$

17. Multiply: $(x - 4)(x^2 + 5x - 3)$

18. The square of a number is subtracted from 60, resulting in -4. Find all such numbers.

19. The product of consecutive integers is 44 more than 14 times their sum. Find all such integers.

20. The length of a rectangle is 1 ft. longer than twice its width. If the area is 78 ft^2, find the rectangle's deimensions.

21. A right triangle has one leg that is 2 ft. longer than the other leg. The hypotenuse is 2 ft. less than twice the shorter leg. Find the lengths of all three sides of hte triangle.

22. Factor: $x^2 + x - 42$

23. Factor: $c^4 - 1$

24. Factor: $-10u^2 + 30u - 20$

25. Factor: $y^3 - 125$

26. Factor: $49 + p^2$

27. Factor: $2x^3 + x^2 - 8x - 4$

28. Factor: $3a^2 + 27ab + 54b^2$

Additional Review (Chapters 5&6) – If Time

1. Write 463,000,000 in scientific notation.

2. Multiply: $\left(2.5 \times 10^{17}\right)\left(6.0 \times 10^{-4}\right)$

3. Divide: $\frac{6.0\ x\ 10^{-4}}{8.0\ x\ 10^{19}}$

4. Simplify each:

a. $5x^0$

b. $(5x)^0$

c. $\left(\frac{-2}{3}\right)^2$

d. -8^{-2}

e. $(-8)^{-2}$

f. $(-3m^2n^3)^4$

g. $\frac{-5x^{-2}y^3}{-10x^4y^{-5}}$

h. $\left(\frac{-3u^{-2}v}{w^{-3}}\right)^{-2} \cdot (uv^{-2})$

5. Given $5x^2 + 3x^7 - 2x + 8$, state find each:

a. Leading coefficient: _____

b. degree of the polynomial: _____

6. Find the degree of the polynomial:

$$x^2y^4z + 3xy^7z^2 - 2x^4yz^3$$

7. Add: $(2x-5)+(4x+8)$

8. Combine:
$(x^2+4x-9)-(2x^2-3x+4)+(5x^2-11)$

9. Multiply each:

a. $(3x-5)(2x+7)$

b. $(4x+7)(4x-7)$

c. $(3x+2y)(4x-9y)$

d. $(5x-8)^2$

e. $(x+5)(x^2-x+2)$

10. Divide: $\frac{5x^2y^3-12xy^4+10x^4y}{8x^3y^2}$

11. Divide: $(6x^2-5x+4)\div(x-3)$

12. Factor: $5x^2+10x+100$

13. Factor: $60xa-30xb-80ya+40yb$

14. Factor: $x^2 - 3x - 28$

15. Solve: $x^2 - 3x - 28 = 0$

16. Factor: $2x^2 - x - 6$

17. Factor: $x^4 - 16$

18. Factor: $x^3 - 27y^3$

19. Solve: $x^2 = 8x$

20. The length of a rectangle is 2 cm more than three times its width. Find the dimensions if the area is 56 cm^2.