

## 7.1 – Introduction to Rational Expressions

Reduce:  $\frac{5}{10}$

Simplify:  $\frac{48a^2b^3}{8a^7b}$

⇒ When simplifying rational expressions, you are actually dividing both the numerator and the denominator by common factors.

Simplify:  $\frac{5x+10}{x+2}$

key phrase: \_\_\_\_\_

Simplify:  $\frac{x^2-4x-32}{x^2-64}$

Simplify:  $\frac{6p^2+12p}{2pq-4p}$

The " - 1" Technique:

Simplify:  $\frac{x-15}{15-x}$

Simplify:  $\frac{16-x^2}{x-4}$

Simplify:  $\frac{49-x^2}{x^2-10x+21}$

Simplify:  $\frac{5}{20a-25}$

Simplify:  $\frac{4t^2+16}{t^4-16}$

## 7.2 – Multiplying and Dividing Fractions

\*Multiply or divide as indicated. Simplify.

$$1. \frac{2p-4}{6p} \cdot \frac{4p^2}{8p-16}$$

$$2. \frac{\frac{p^2-6p+8}{24}}{\frac{16-p^2}{6p+6}}$$

$$3. \frac{6a-6}{a^2+6a+5} \cdot \frac{a^2+5a}{12a}$$

$$4. \frac{x^2-xy-2y^2}{x+2y} \div \frac{x^2-4xy+4y^2}{x^2-4y^2}$$

## 7.3 & 7.4 – Adding and Subtracting Fractions

Recall... when adding and subtracting fractions, you need to have a \_\_\_\_\_.

$$\text{Add: } \frac{7}{8} + \frac{3}{5}$$

\*Find the least common multiple between each. Also state how many times each expression goes into the least common multiple.

$$1. 5x^2y^3 \text{ and } 6x^4y$$

$$2. 3x + 6 \text{ and } x^2 - 4$$

$$3. 5x - 10 \text{ and } 10x + 30$$

4.  $x^2 - x - 6$  and  $3x - 9$

5.  $x + 2$  and  $x + 4$

\*Add or subtract. Simplify if possible.

6.  $\frac{5}{x-2} + \frac{3}{x-2}$

7.  $\frac{x+3}{x+4} + \frac{x+5}{x+4}$

8.  $\frac{x+7}{x-3} - \frac{x+5}{x-3}$

9.  $\frac{4}{3x-15} - \frac{x}{5-x}$

10.  $\frac{4}{5xy^3} + \frac{2x}{15y^2}$

11.  $\frac{6}{y-1} + \frac{9}{y}$

12.  $\frac{4n}{n-8} - \frac{2n-1}{8-n}$

13.  $\frac{6a}{a^2-b^2} + \frac{2a}{a^2+ab}$

$$14. \frac{x+2}{3x+5} - \frac{3x+7}{3x+5}$$

$$15. \frac{5}{6x^3y^4z} + \frac{8}{9x^2y^3z^4}$$

$$16. \frac{4}{x^2-9} + \frac{3}{2x+6}$$

$$17. \frac{5}{4x-8} - \frac{3}{3x-6}$$

$$18. \frac{3x}{x^2+x-6} + \frac{x}{x^2+5x+6}$$

$$19. \frac{2n}{3n^2-8n-3} + \frac{1}{6-2n} - \frac{2}{3n+1}$$

## 7.5 – Complex Fractions

1. 
$$\frac{\frac{2x-10}{4}}{\frac{x^2-5x}{3x}}$$

2. 
$$\frac{2 + \frac{1}{x}}{4 + \frac{1}{x}}$$

3. 
$$\frac{\frac{1}{m^2} + \frac{2}{3}}{\frac{1}{m} - \frac{5}{6}}$$

4. 
$$\frac{\frac{m}{7} - \frac{7}{m}}{\frac{1}{7} + \frac{1}{m}}$$

5. 
$$\frac{\frac{2}{p} + \frac{p}{2}}{\frac{p}{3} - \frac{3}{p}}$$

6. 
$$\frac{6 + \frac{6}{k}}{1 + \frac{1}{k}}$$

Review of Sections 7.1 – 7.5

1. Simplify:  $\frac{x^2+5x}{x^2-25}$

2. Multiply:  $\frac{x^2-x-6}{3x-9} \cdot \frac{5x+15}{10x+20}$

3. Add:  $\frac{x+7}{x-1} + \frac{4}{2x-2}$

4. Subtract:  $\frac{x-8}{x+4} - \frac{x+6}{x+4}$

5. Add:  $\frac{3}{x^2-2x-24} + \frac{5}{x^2-16}$

6. Add:  $\frac{x+1}{x+2} + \frac{x+3}{x+4}$

7. Simplify:  $\frac{\frac{x}{x+2} - \frac{2}{x-2}}{\frac{x}{x+2} + \frac{2}{x-2}}$

## 7.6 – Rational Equations

**Main Idea:** Clear out the fractions by multiplying through on each side of the equation by the least common denominator.

Be careful to check that your solution(s) work. Make sure that when you substitute your solution into the problem, you don't get a zero in the denominator.

\*Solve each of the following equations.

$$1. \frac{4}{x} = \frac{3}{x} + \frac{1}{8}$$

$$2. \frac{x}{12} + \frac{x+3}{3x} = \frac{1}{x}$$

$$3. \frac{2}{4n-4} - \frac{7}{4} = \frac{-3}{n-1}$$

$$4. \frac{4x}{x+3} - \frac{12}{x-3} = \frac{4x^2+36}{x^2-9}$$

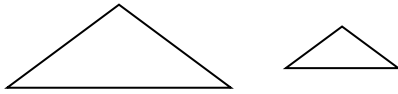
$$5. \frac{x^2+3x}{x-1} = \frac{4}{x-1}$$

$$6. \frac{y+1}{2y} = \frac{2}{3}$$

### 7.7 – Proportions and Word Problems

#### A. Geometrical Figures

Similar triangles:



At the same time that a yardstick casts a 6 ft shadow, a tree casts a 30 ft shadow. How tall is the tree?

#### B. Proportional Word Problems

Key phrases: \_\_\_\_\_  
or \_\_\_\_\_.

Key idea: Line up the units (either vertically or horizontally).

7.7 #26 Tim takes his pulse for 10 seconds and counts 12 beats. How many beats per minute is this?

A map has a scale of 75 mi/in. If two cities measure 3.5 inches apart, how many miles does this represent?

#### C. Distance Problems



7.7 #41 A boat travels 54 miles upstream in the same time it takes to travel 66 miles downstream. If the current is 2 mph, what is the boat's speed in still water?

7.7 #46 Devon can cross-country ski 5 km/hr faster than his sister Shanelle. Devon skis 45 km in the same time Shanelle skis 30 km. Find their speeds.

#### D. Work Problems

Fred can paint his house in 30 hours. Barney can paint the same house in 50 hours. How long would it take if they worked together?

Machine A can copy a report in 6 minutes. Machine B can copy the same report in 8 minutes. How long would it take if they worked together?

## 7.8 – Variation

Direct Variation

Inverse Variation

"k" is called the

### Process for Solving:

1. Identify the type of variation. Write the "base" equation.
2. Substitute in the given values to solve for k.
3. Write the general variation equation for the problem using the value of k you just found.
4. Use the clues to find the requested value.

1.  $y$  varies directly with  $x$ , and  $y=10$  when  $x=2$ . Find the general variation equation and use it to find  $y$  when  $x=14$ .

2.  $y$  varies inversely with  $x$ , and  $y=6$  when  $x=10$ . Find the general variation equation and use it to find  $y$  when  $x=8$ .

3.  $y$  varies jointly with  $x$  and  $w$  and inversely with  $z$ .  $y=4$  when  $x=3$ ,  $z=8$  and  $w=12$ . Find  $y$  when  $x=6$ ,  $z=10$  and  $w=8$ .

4.  $I$  varies inversely with the square of  $d$ , and  $I=30$  when  $d=4$ . Find  $I$  when  $d=6$ .

5. The number of turkeys needed for a Thanksgiving dinner is directly proportional to the number of guests. If a cook knows that 3 turkeys will feed 42 guests, how many turkeys should he cook for 70 guests?

## Some Chapter 7 Review Problems

1. What values of  $x$  are NOT in the domain of

$$g(x) = \frac{x+9}{x^2-2x-24}?$$

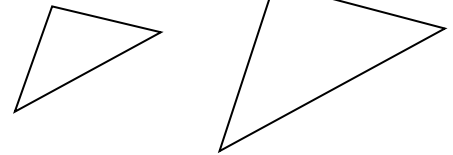
2. Add:  $\frac{5}{x-3} + \frac{2}{x}$

3. Simplify:  $\frac{\frac{6}{x^2} - \frac{2}{3x}}{\frac{4}{x} - \frac{2}{x^2}}$

4. Solve:  $\frac{x+1}{3} - \frac{x-1}{6} = \frac{1}{6}$

5. Solve:  $\frac{1}{x+2} = \frac{4}{x^2-4} - \frac{1}{x-2}$

6. Find  $x$  and  $y$ .



7. Hose A can fill a pool in 10 hours. Hose B can fill it in 12 hours. How long would it take if both hoses worked together?

8. The current in Lazy River is 2 mph. If one motors upstream 12 miles in the time it takes to motor 20 miles downstream, what is the speed of the boat in still water?

9. If  $T$  varies inversely as  $x$  and when  $x=40$ ,  $T=200$ , find  $T$  when  $x=50$ .

10. Fred and Barney can complete a job in 12 hours if working together. If Fred works twice as fast as Barney, how long would each take, working alone?

11. Bud goes 10 mph faster on his Harley than Ed goes on his Honda motorcycle. If Bud travels 105 miles in the same time that Ed travels 90 miles, what are the rates of the two bikers?

12. Simplify:  $\frac{7-2x}{2x-7}$

13. Add:  $\frac{2}{y^2+4y+3} + \frac{1}{3y+9}$

14. Subtract:  $\frac{x}{x-2} - \frac{8}{x^2-4}$

15. Solve  $\frac{V}{h} = \frac{\pi r^2}{3}$  for h.

16. Simplify:  $\frac{1 - \frac{4}{m}}{m - \frac{16}{m}}$

17. Solve:  $\frac{3}{c-2} - \frac{1}{c+1} = \frac{7}{c^2-c-2}$

18. There are two machines that produce aluminum cans. The newer machine can produce 4200 cans in 210 minutes. It takes the older machine 280 minutes to produce that many cans. If the two machines work together, how long will it take them to produce 4200 cans?

### 8.1 – Introduction to Radicals

$$\sqrt{25} = \underline{\hspace{2cm}} \text{ since } \underline{\hspace{2cm}}.$$

Question:

Answer:

Evaluate:  $\sqrt{-25}$

Evaluate:  $-\sqrt{25}$

Evaluate:  $\sqrt[3]{-8}$

Evaluate:  $\sqrt{(-8)^2}$

Property:

\*Simplify each of the following:

1.  $\sqrt{100}$

2.  $\sqrt{144x^2}$

3.  $\sqrt{225w^4}$

4.  $\sqrt{169w^6}$

5.  $\sqrt[4]{x^4}$

When, exactly, do you use absolute values?

### 8.2 – Simplifying Radicals

\*Evaluate or Simplify each:

1.  $\sqrt{50}$

2.  $\sqrt{108}$

3.  $\sqrt[3]{54}$

4.  $\sqrt{98a^5b^7}$

5.  $\sqrt[5]{32x^{31}y^{87}}$

6.  $\sqrt{\frac{25}{36}}$

7.  $\sqrt[4]{(-4)^4}$

8.  $\sqrt{60x^3}$

9.  $\sqrt[3]{16x^2y^5}$

Review. Simplify each.

1.  $\sqrt{-64}$

2.  $-\sqrt{64}$

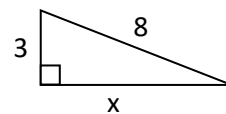
3.  $\sqrt{(-8)^2}$

4.  $\sqrt[3]{-27}$

5.  $\sqrt{48x^5y^7}$

6.  $\sqrt[11]{x^{56}y^{90}}$

7. Find x:



### 8.3 – Adding and Subtracting Radicals

⇒ Property:  $\sqrt[n]{a} + \sqrt[n]{a} = 2\sqrt[n]{a}$

\*Combine. Simplify if possible.

1.  $5\sqrt{7} + 9\sqrt{7}$

2.  $\sqrt{2} + 2\sqrt{2} + 3\sqrt{2}$

3.  $5\sqrt{3} + 4\sqrt{2} + 7\sqrt{3} - 9\sqrt{2}$

4.  $4\sqrt{7} + 3\sqrt{10} + 5\sqrt{7} - 9$

5.  $7\sqrt{8} - 9\sqrt{50}$

6.  $8\sqrt{27} + 5\sqrt{12} - 2\sqrt{75}$

7.  $8y\sqrt{15} - 3y\sqrt{15}$

8.  $11\sqrt{a^4b} - a^2\sqrt{b} - 9a\sqrt{a^2b}$

9.  $3\sqrt{20x} - 7\sqrt{45x}$



### 8.4 – Multiplying Radicals

⇒ Property:  $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$

\*Multiply. Simplify, if possible.

1.  $\sqrt{3} \cdot \sqrt{2}$

2.  $5\sqrt{2}(7\sqrt{11})$

3.  $7x\sqrt{5}(3x\sqrt{10})$

4.  $3a^2b\sqrt{6ab}(2ab\sqrt{3a})$

5.  $(4\sqrt{2})^2$

6.  $3\sqrt{2}(4\sqrt{2} + 7)$

7.  $(\sqrt{3} - \sqrt{2})(2\sqrt{3} - \sqrt{2})$

8.  $(4\sqrt{3} - 2\sqrt{5})(5\sqrt{3} + 4\sqrt{5})$

9.  $(3\sqrt{a} - 5)^2$

10.  $(5\sqrt{c} + 2\sqrt{d})^2$

## 8.5 – Dividing Radicals

Main idea: An expression is not considered to be simplified if it contains a radical in its denominator. The process of removing the radical from the denominator is called

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\*Simplify each.

1.  $\frac{\sqrt{64}}{\sqrt{81}}$

2.  $\frac{\sqrt{13}}{\sqrt{25}}$

3.  $\sqrt{\frac{21}{7}}$

4.  $\frac{\sqrt{15}}{\sqrt{5}}$

5.  $\frac{5}{\sqrt{2}}$

6.  $\frac{6}{\sqrt{3}}$

7.  $\frac{\sqrt{x}}{\sqrt{32}}$

8.  $\frac{14}{\sqrt{7}}$

9.  $\frac{\sqrt[3]{24x^5}}{\sqrt[3]{3x^4}}$

10.  $\frac{2+\sqrt{3}}{3+\sqrt{3}}$

11.  $\frac{2\sqrt{3}-3\sqrt{2}}{3\sqrt{3}+2\sqrt{2}}$

12.  $\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$

13. Simplify:  $\frac{4\sqrt{6}+8\sqrt{2}}{12}$

## 8.6 – Solving Radical Equations

### Steps:

1. Isolate the radical.
2. Square both sides.
3. Check your solutions!

\*Solve each:

1.  $\sqrt{x+2} = 5$

2.  $\sqrt[3]{x-4} = 2$

3.  $\sqrt{x+5} = -2$

4.  $\sqrt{x^2+5x} = \sqrt{150}$

5.  $\sqrt{9y^2-8y+1} = 3y+1$

6.  $\sqrt{6t+7} = t+2$

7.  $\sqrt{2m+1} + 7 = m$

9.  $\sqrt{x+13} = \sqrt{x-2} + 3$

8. The square root of twice a number is 4 less than the number. Find the number.

10.  $\sqrt{x} = \sqrt{x+5} + 1$

The following problem is a bit more challenging:

11. Solve:  $\sqrt{-15w - 31} - \sqrt{3w + 32} = -3$

Chapter 8 Review

1. Simplify:  $6\sqrt{40x^3y^2}$

2. Combine:  $5\sqrt{3} + 6\sqrt{3} - 7\sqrt{2}$

3. Multiply:  $5\sqrt{3}(2\sqrt{6})$

4. Multiply:  $(3 + 2\sqrt{3})(2 - 4\sqrt{3})$

5. Simplify:  $(2\sqrt{3} + 5\sqrt{2})^2$

6. Divide:  $\frac{\sqrt{21}}{\sqrt{3}}$

7. Divide:  $\frac{\sqrt{10}}{2\sqrt{7}}$

8. Divide:  $\frac{5-2\sqrt{2}}{4+3\sqrt{2}}$

9. Solve:  $\sqrt{x-4} = 7$

10. Solve:  $\sqrt{x-1} = x-3$

## 9.1 – Solving Quadratic Equations

### A. Factorable Quadratic Equations

These problems are just like those at the end of chapter 6.

\*Solve each equation.

1.  $y^2 - 2y - 35 = 0$

2.  $(x + 2)(x + 6) = 5$

3.  $x^2 = 25$

### B. Taking the Square Root of each Side

4.  $x^2 = 30$

5.  $(x + 5)^2 = 11$

6.  $(x - 3)^2 = 12$

7.  $(5x - 2)^2 + 7 = 13$

8.  $7(3x + 1)^2 - 5 = -2$

## 9.2 – Completing the Square

### Process:

1. Write your equation in the form:
2. If there's a leading coefficient, divide both sides by it.
3. Cut the number in front of  $x$  in half; write this new value on the line below.
4. Square this new value and write the product in the blank on the line above; add this product to the right side of the equation also.
5. Insert an  $x$ , parentheses and exponent on the left to complete the square.
6. Add together the values on the right side.
7. Square root each side of the equation. Don't forget the plus-or-minus sign!
8. Solve for  $x$ .

\*Solve each by completing the square:

1.  $x^2 + 8x - 3 = 0$

2.  $x^2 + 10x + 4 = 0$



3.  $x^2 - 12x - 7 = 0$

4.  $x^2 + 5x - 2 = 0$

5.  $x^2 - 3x - 1 = 0$

6.  $3x^2 + 12x - 5 = 0$

7.  $2x^2 - 11x - 7 = 0$

8.  $ax^2 + bx + c = 0$

\*Find the value of k needed in order to complete the square:

9.  $x^2 + 10x + k$

10.  $x^2 + kx + 9$

### 9.3 – The Quadratic Formula

\*In each equation, identify the values of a, b and c:

1.  $x^2 + 5x - 2 = 0$

2.  $-3x^2 + 7x = -4$

\*Solve each equation using the quadratic formula.

3.  $3x^2 + 5x - 1 = 0$

4.  $4x^2 + 2x - 5 = 0$

5.  $3x^2 - x + 7 = 0$

6.  $x^2 + 6x - 2 = 0$

Summary:  $ax^2 + bx + c = 0$

a. If  $c = 0$ , use factoring.

b. If  $b = 0$ , square root each side.

c. If  $a = 0$ , it's not quadratic!

Solve  $x^2 - 2x - 15 = 0$  in two ways.

## Chapter 9 Review:

\*Solve each of the following equations:

1.  $2x^2 + 12x = -10$

2.  $8x^2 - 8x = 0$

3.  $x^2 + 4x + 4 = 7$

4.  $4x^2 + 4x = 5$

5.  $x^2 - 16 = 0$

6.  $(x + 1)^2 = 18$

$$7. \left(x + \frac{3}{2}\right)^2 = \frac{7}{2}$$

$$8. (x - 13)^2 = 64$$

$$9. 4x^2 + 12x = 7$$

$$10. \frac{5}{x-3} - \frac{30}{x^2-9} = 1$$

$$11. (2x - 3)^2 + 5 = 8$$

12. Solve:  $2w^2 - w + 66 = (w - 6)^2$

13. Solve:  $(x + 4)^2 = 2x^2 - 3x + 46$

Final Exam Review

1. A 5% acid mixture is to be combined with a 40% acid solution. How much of each should be used to make 20 liters of a 25% acid solution?

2. Find three consecutive odd integers who sum is 51.

3. The product of consecutive integers is 72. Find all such numbers.

4. The length of a rectangle is 3 more than twice its width. Find the dimensions if the area is 65 square units.

5. The length of a rectangle is 1 cm less than twice its width. Find the dimensions if the perimeter is 22 cm.

6. Half a number is 18 less than twice the number. Find the number.

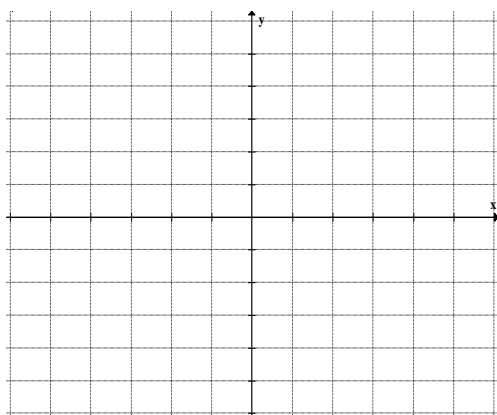
7. Find the x- and y-intercepts of  $4x - 7y = 28$ .

8. Find the slope of  $5x - 3y = 11$ .

9. Find the slope between  $(-1,7)$  and  $(2,5)$ .

10. Find the equation of the line which passes through  $(-4,1)$  and is perpendicular to the line  $y = \frac{4}{7}x - 13$ .

11. Graph:  $y < \frac{-2}{3}x + 4$



12. Solve:  $20 - (3 + 4x) \leq 2(x - 7) + 1$

13. Evaluate:  $5^{-2}$

14. Simplify:  $(-4x^3y^{-2})^{-2}$

15. Use the commutative property to finish this statement:  $8 + 7 = \underline{\hspace{2cm}}$

16. Use the associative property to finish this statement:  $a(ce) = \underline{\hspace{2cm}}$

17. Which property is illustrated?  
 $5 + (-5) = 0$

18. Simplify:  $\frac{8x-16}{x^2-4}$

19. Add:  $\frac{5}{x} + \frac{3}{x-2}$



20. Simplify:  $\sqrt{\frac{50x}{y^2z}}$

21. Find the slope of the line perpendicular to  $y = 5$ .

22. Combine:  $3\sqrt{50} - 4\sqrt{18}$

23. Solve:  $\begin{cases} y = x - 2 \\ 4x - y = 17 \end{cases}$

24. Bob bought two computers, one desktop and one laptop. Before finance charges, the laptop cost \$500 less than the desktop. Bob paid for the computers using two different financing plans. For the desktop the interest rate was 3% per year, and for the laptop it was 5% per year. The total finance charges for one year were \$335. How much did the desktop computer cost before finance charges?