

Lecture Guide

Math 50 - Elementary Algebra

to accompany

Introductory Algebra, 2nd edition

Miller, O'Neil, Hyde

Prepared by

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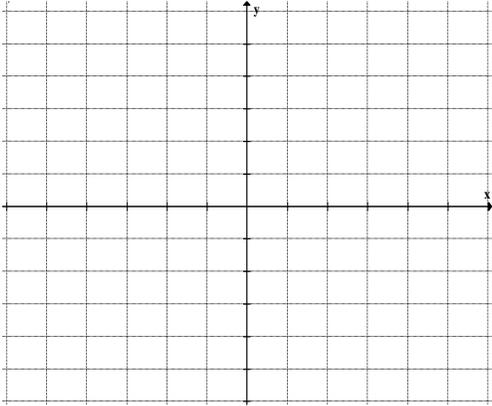
Victor Valley College

Last updated: 10/8/12

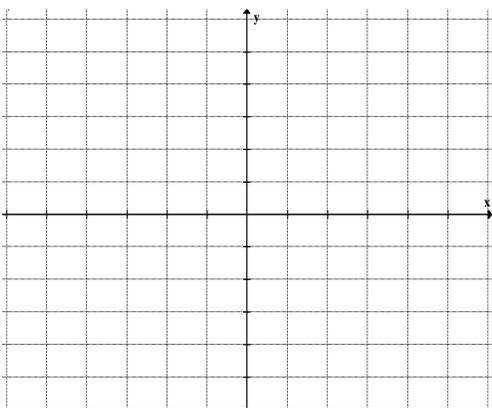
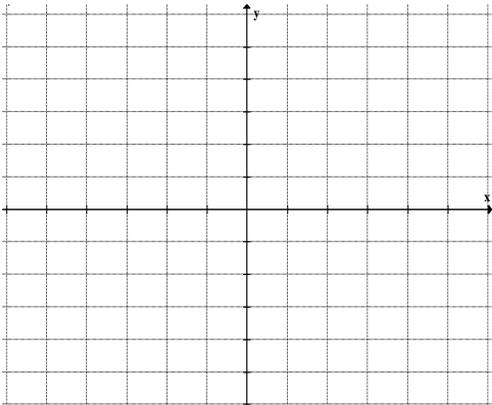
3.1 – The Rectangular Coordinate System

Introduction to Graphing

The four quadrants:



(x,y) coordinates:



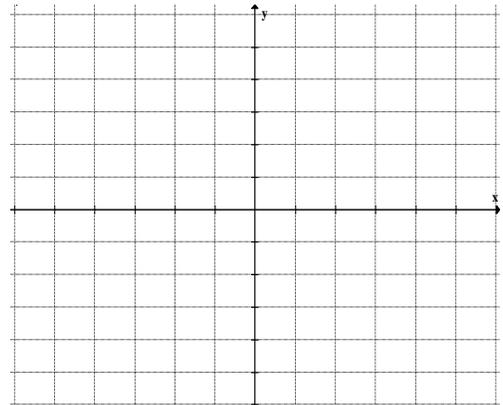
3.2 – Linear Equations in Two Variables

A line is an infinite collection of (x,y) points, each of which satisfy the equation of that line.

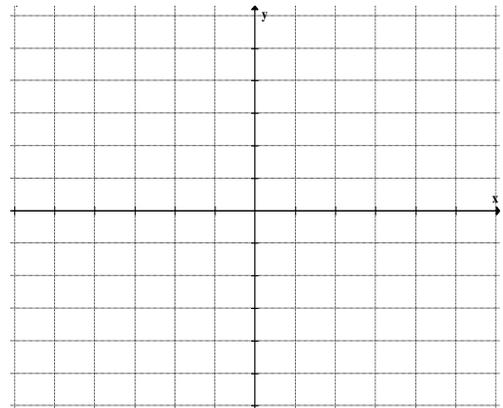
*Is (3, -4) on the line $6x + 5y = -2$?

*Graph each line:

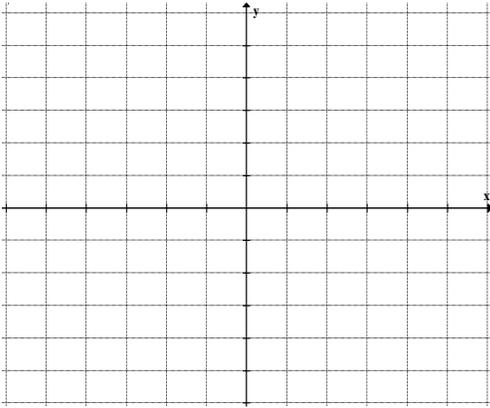
1. $y = x + 5$



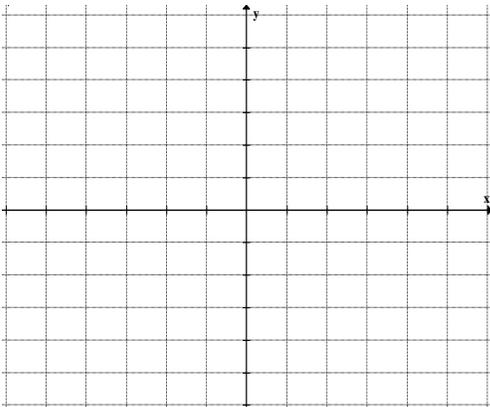
2. $y = -x + 2$



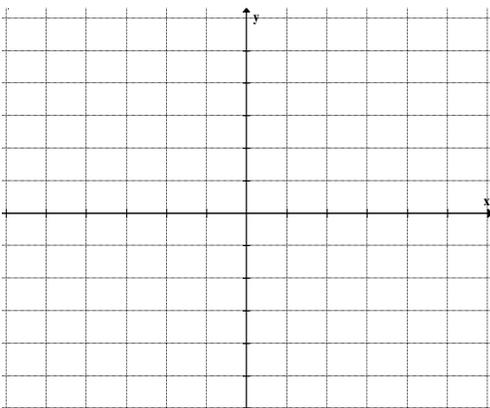
3. $y = 2x + 1$



4. $4x + 3y = 12$

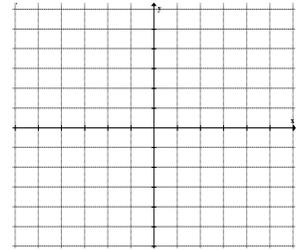
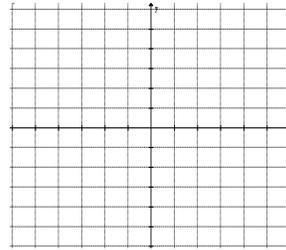


5. $2x - 3y = 6$

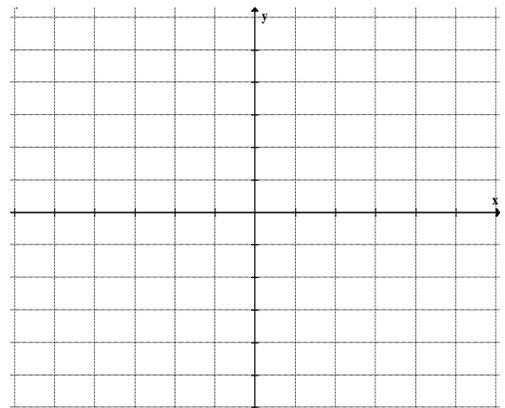


Summary:

- a. When the given line is of the form $y = \underline{\hspace{2cm}}$, use an x-y chart with any choices of x (usually $x=0,1,2$ is best).
- b. When the given line is of the form $ax + by = c$, also known as standard form, graph using intercepts.
- c. When the given line is missing a variable, it is either horizontal or vertical.



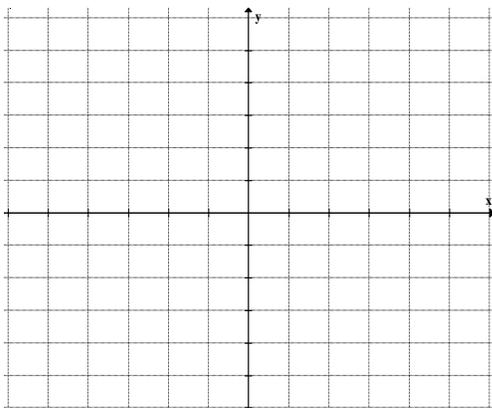
6. Graph $y = \frac{1}{2}x + 3$



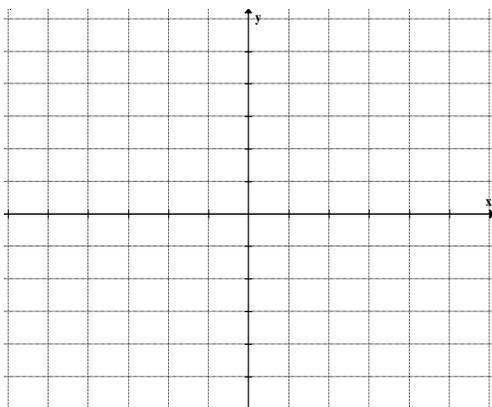
7. Find the x- and y-intercepts of $4x - 7y = 16$.

Review:

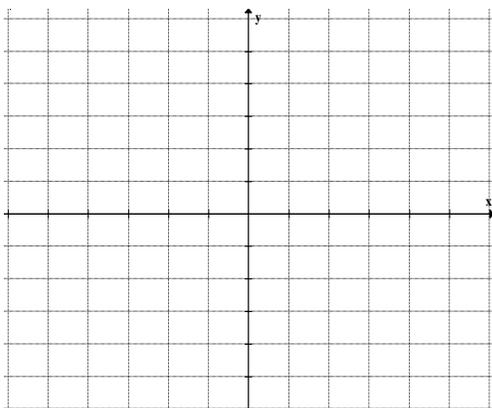
8. Graph $4x + 3y = -12$



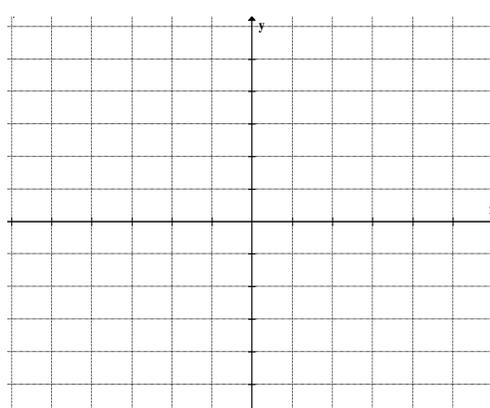
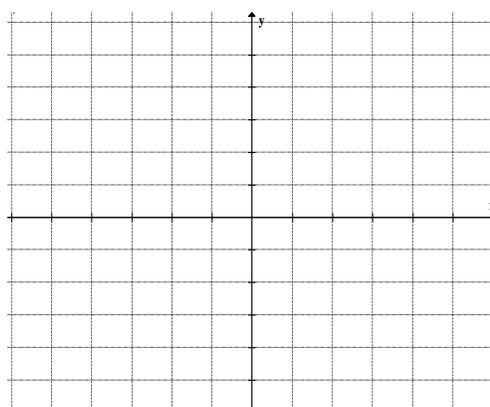
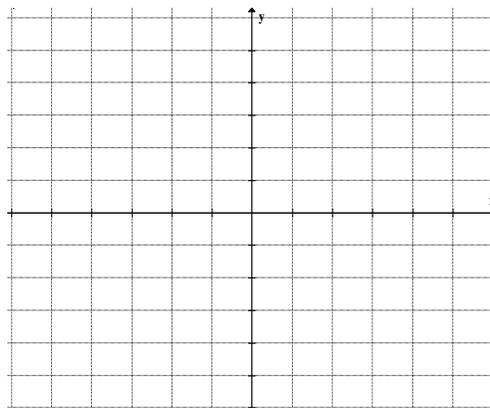
9. Graph $x = 3$



10. Graph $y = 3x - 1$



Extra Practice:



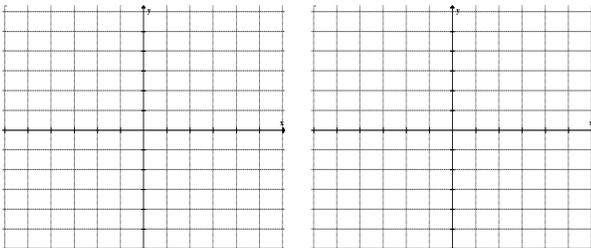
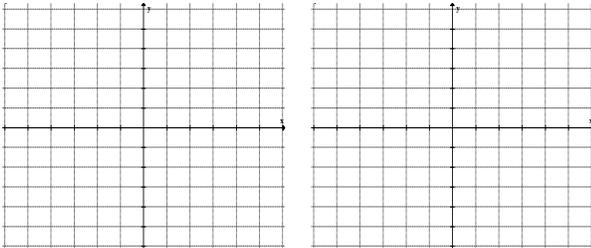
3.3 – Slope of a Line and Rate of Change

Slope:

In word problems, slope is sometimes referred to as a rate of change.

Formula: $m = \frac{\text{rise}}{\text{run}} =$

*Find the slope of each line:

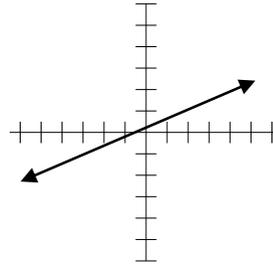


*Find the slope of the line which runs through the given pair of points:

1. $(-2,3)$ and $(4,1)$

2. $(4,5)$ and $(11,-3)$

*Find the slope of the graphed line (passes through $(-3,-1)$ and $(2,1)$):



**Parallel lines have the _____ slope.

**The slopes of perpendicular lines are _____
_____ of each other.

examples:

$y = mx + b$ is called the _____
form of a line.

*In each equation, identify the slope and the y-intercept.

1. $y = 3x - 5$

2. $y = \frac{1}{3}x - \frac{4}{7}$

3. $y = 5x$

4. $y = x - 9$

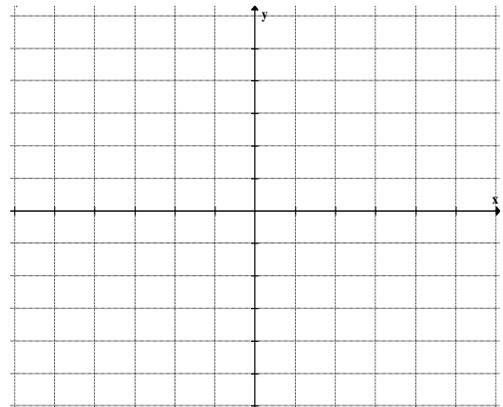
5. $3x + 7y = 11$

6. $2x + 5y = 3$

*Are the lines between these pairs of points parallel, perpendicular, or neither?

line 1: (1,9) and (0,4) ; line 2: (5,2) and (10,1)

*Graph the line which passes through $(-2, -3)$, having a slope of $\frac{3}{4}$.



*Name two additional points which are on the line you just graphed.

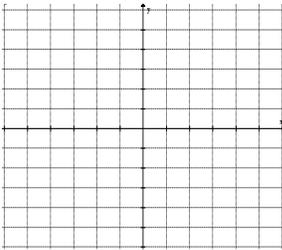
3.4 – Slope-Intercept Form of a Line

Graphing Using the Slope Intercept Form of a Line

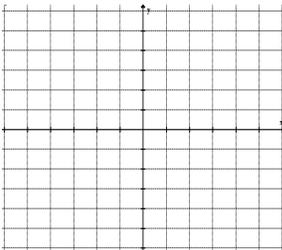
- A. Identify and plot the y-intercept.
- B. Identify the slope. Rise and run (to the right) from the y-intercept to another point on the line.
- C. Draw the line.

*Graph each of the following lines:

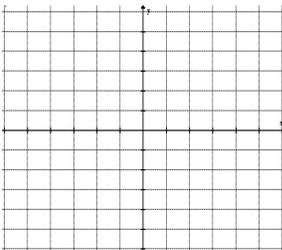
1. $y = \frac{2}{5}x - 3$



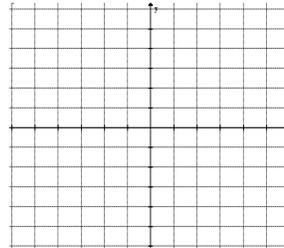
2. $y = \frac{-1}{3}x + 2$



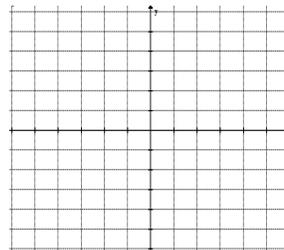
3. $y = -x + 3$



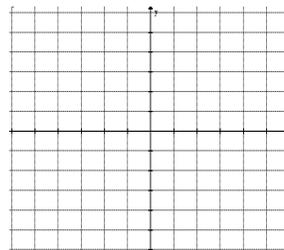
4. $y = -2$



5. $y = x$



6. $-3x + 4y = 12$



*Are the following pairs of lines parallel, perpendicular, or neither?

7.
$$\begin{cases} y = \frac{3}{5}x + 2 \\ y = \frac{-3}{5}x + 7 \end{cases}$$

8.
$$\begin{cases} y = x + 4 \\ y = -x + 7 \end{cases}$$

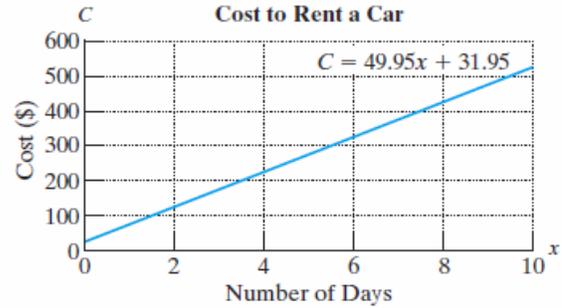
*Find the equation of the line having the given attributes:

9. $m = \frac{3}{11}$; y-intercept (0,8)

10. passes through (0, -3) with slope of -2.

3.4 #79 The cost for a rental car is \$49.95 per day plus a flat fee of \$31.95 for insurance. The equation, $C = 49.95x + 31.95$ represents the total cost, C (in dollars), to rent the car for x days.

a. Identify the slope. Interpret the meaning of the slope in the context of this problem.



b. Identify the C-intercept. Interpret the meaning of the C-intercept in the context of this problem.

c. Use the equation to determine how much it would cost to rent the car for 1 week.

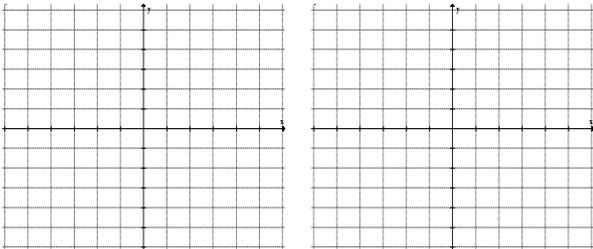
****Suggested Assignment: pg. 252 #1-20**

Review

1. Solve $m(2 - x) = 7 - y$ for x .

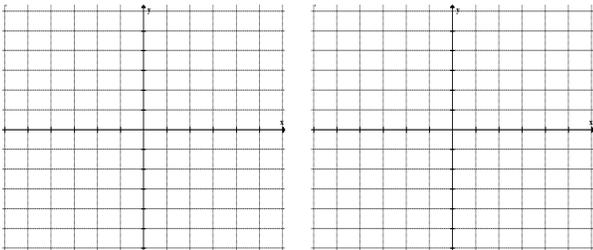
2. In which quadrant(s) is $x > 0$ and $y > 0$?

3. In which quadrant(s) is $xy > 0$?

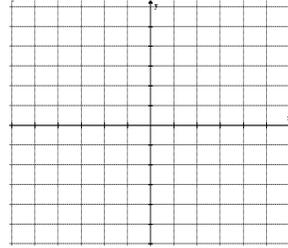


4. Graph $x = 2$

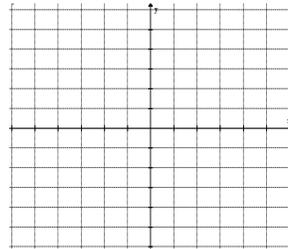
5. Graph $y = 2x - 5$



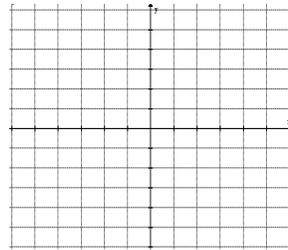
6. Graph on the same graph:
$$\begin{cases} y = \frac{1}{2}x + 2 \\ y = \frac{1}{2}x - 1 \end{cases}$$



7. Graph $y = \frac{-2}{3}x + 1$



8. Graph the line passing through $(2,3)$ with $m = \frac{-1}{2}$.



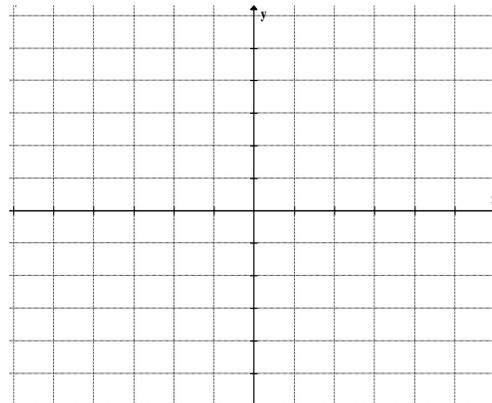
9. Write the equation of each line below:

10. Convert $4x - 5y = 8$ into slope-intercept form.

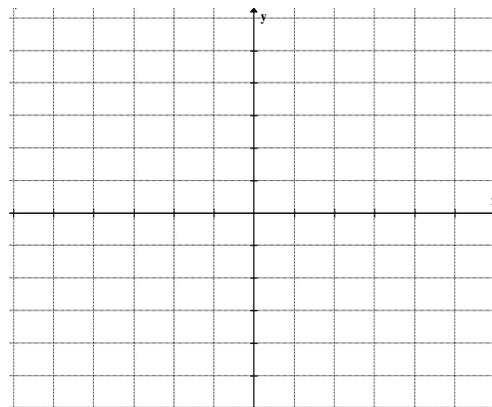
11. Convert $-2x - 8y = 7$ into slope-intercept form.

12. Find the slope between $(-2,7)$ and $(3,5)$.

13. Graph $y = |x + 2| - 1$



14. Graph $y = x^2$



3.5 – Point-Slope Formula

Three Forms of a Line

1. $ax + by = c$

2. $y = mx + b$

3. $y - y_1 = m(x - x_1)$

*Find the equation of the line (in slope-intercept form) which has the following characteristics.

a. $m = \frac{2}{3}$ and has y-intercept (0,5).

b. $m = \frac{-3}{5}$ and the line passes through (-1,4).

c. passes through (-2,5) and (3,7).

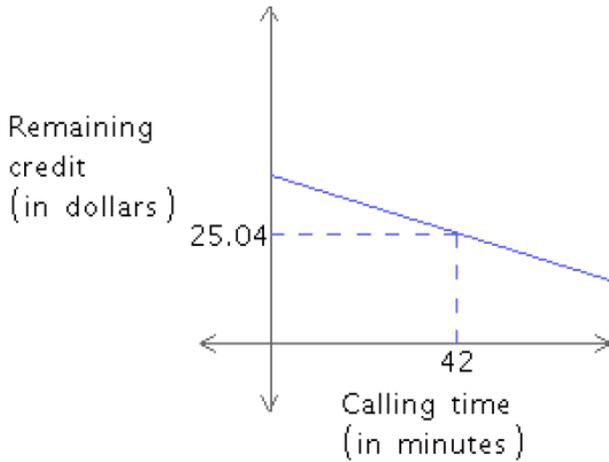
d. passes through (-5,1) and is parallel to the line $3x - 4y = 11$.

e. passes through (-6,2) and is perpendicular to the line $y = \frac{-4}{3}x + 7$.

Aleks Problem:

Suppose that the credit remaining on a phone card (in dollars) is a linear function of the total calling time (in minutes). When graphed, the function gives a line with a slope of -0.12 . See the figure below.

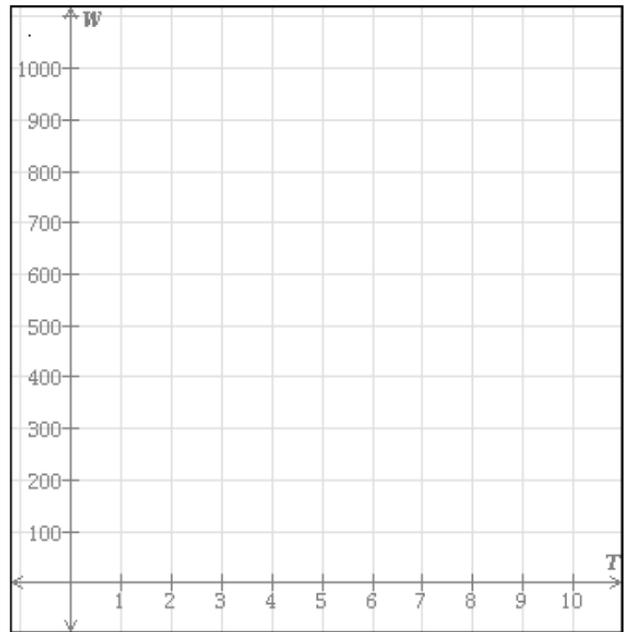
There is \$25.04 in credit remaining on the card after 42 minutes of calls. How much credit was there after 29 minutes of calls?



Aleks Problems:

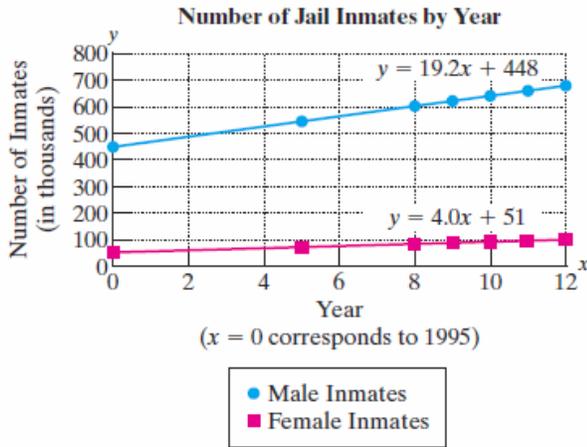
Owners of a recreation area are filling a small pond with water. They are adding water at a rate of 35 liters per minute. There are 700 liters in the pond to start.

Let W represent the amount of water in the pond (in liters), and let T represent the number of minutes that water has been added. Write an equation relating W to T , and then graph your equation using the axes below.



3.6 – Applications of Linear Equations and Modeling

3.6 #10 The graph depicts the rise in the number of jail inmates in the United States since 1995. Two linear equations are given: one to describe the number of female inmates and one to describe the number of male inmates by year.



(Source: U.S. Bureau of Justice Statistics)

Let y represent the number of inmates (in thousands).
Let x represent the number of years since 1995.

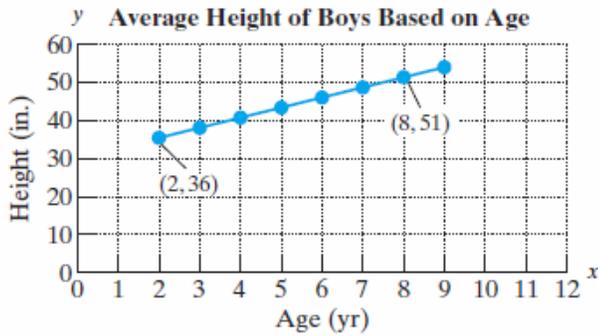
- What is the slope of the line representing the number of female inmates? Interpret the meaning of the slope in the context of this problem.
- What is the slope of the line representing the number of male inmates? Interpret the meaning of the slope in the context of this problem.
- Which group, males or females, has the larger slope? What does this imply about the rise in the number of male and female prisoners?

- Assuming this trend continues, use the equation to predict the number of female inmates in 2015.

3.6 #14 For a recent year, children's admission to the Minnesota State Fair was \$8. Ride tickets were \$0.75 each. The equation $y = 0.75x + 8$ represented the cost, y , in dollars to be admitted to the fair and to purchase x ride tickets.

- Determine the slope of the line represented by $y = 0.75x + 8$. Interpret the meaning of the slope in the context of this problem.
- Determine the y -intercept. Interpret its meaning in the context of this problem.
- Use the equation to determine how much money a child needed for admission and to ride 10 rides.

3.6 #17 The graph shows the average height for boys based on age. Let x represent a boy's age, and let y represent his height (in inches).

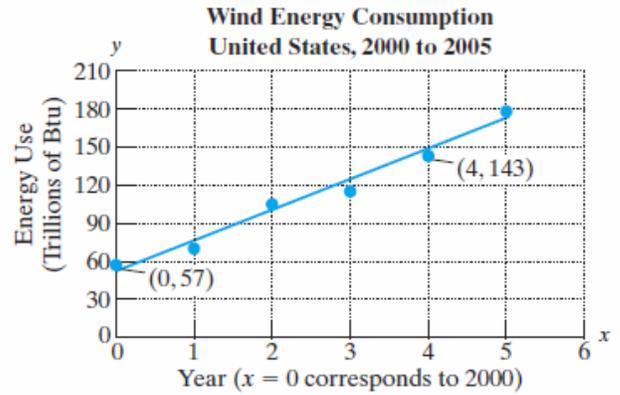


(Source: National Parenting Council)

a. Find a linear equation that represents the height of a boy versus his age.

b. Use the linear equation found in part (a) to predict the average height of a 5-year-old boy.

3.6 #18 Wind energy is one type of renewable energy that does not produce dangerous greenhouse gases as a by-product. The graph shows the consumption of wind energy in the United States for selected years. The variable y represents the amount of wind energy in trillions of Btu, and the variable x represents the number of years since 2000.



(Source: United States Department of Energy)

a. Use the points $(0, 57)$ and $(4, 143)$ to determine the slope of the line.

b. Interpret the slope in the context of this problem?

c. Use the points $(0, 57)$ and $(4, 143)$ to find a linear equation relating the consumption of wind energy, y , to the number of years, x , since 2000.

d. If this linear trend continues beyond the observed data values, use the equation in part (c) to predict the consumption of wind energy in the year 2010.

3.6 #20 A phone bill is determined each month by a \$18.95 flat fee, plus \$0.08 per minute of long distance.

- a. Write a linear equation to compute the monthly cost of a phone bill, y , if x minutes of long distance are used.

- b. Use the equation to compute the phone bill for a month in which 1 hr and 27 min of long distance was used.

3.6 #22 An air-conditioning and heating company has a fixed monthly cost of \$5000. Furthermore, each service call costs the company \$25.

- a. Write a linear equation to compute the total cost, y , for 1 month if x service calls are made.

- b. Use the equation to compute the cost for 1 month if 150 service calls are made.

3.6 #24 A beverage company rents a booth at an art show to sell lemonade. The daily cost to rent a booth is \$35. Each lemonade costs \$0.50 to produce.

- a. Write a linear equation to compute the total cost, y , for 1 day if x lemonades are produced.

- b. Use the equation to compute the cost for 1 day if 350 lemonades are produced.

Aleks Problem:

Consider $y = \frac{2}{3}x + 7$.

- a. Find the line parallel to this line through (4,1).
- b. Find the line perpendicular to this line through (4,1).

4.1 – Solving Systems by the Graphing Method

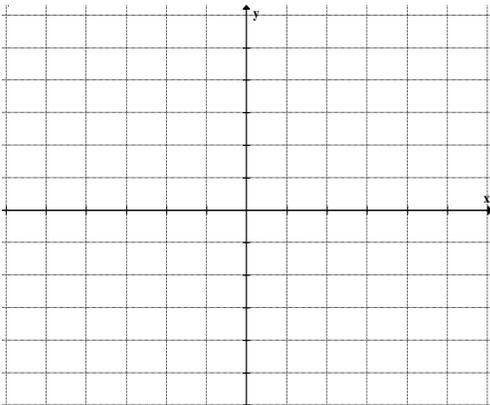
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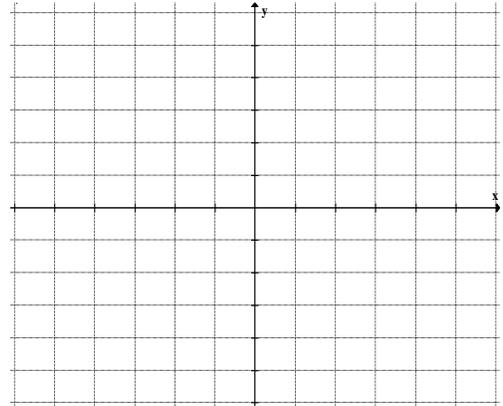
Three Cases:

*Find the solution to each system by graphing.

$$1. \begin{cases} y = -2x \\ y = -3x + 1 \end{cases}$$



$$2. \begin{cases} 2x + y = 4 \\ 4x - 2y = -4 \end{cases}$$



Aleks Problem:

For each point in the table below, decide whether it is on Line 1, Line 2, both, or neither.

$$\text{Line 1: } y = 2x - 17$$

$$\text{Line 2: } 13x + 6y = 23$$

(x, y)	Line 1	Line 2	Both Lines	Neither Line
$(4, 5)$	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
$(6, -5)$	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
$(5, -7)$	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>

4.2 – Solving Systems using Substitution

1. Solve:
$$\begin{cases} 3x - 7y = -2 \\ y = -2x + 27 \end{cases}$$

2. Solve:
$$\begin{cases} 2y = x + 9 \\ y = -3x + 1 \end{cases}$$

3. Solve:
$$\begin{cases} x + y = 1 \\ 2x - y = -2 \end{cases}$$

4. Solve:
$$\begin{cases} y = 0.75x - 3 \\ -3x + 4y = -12 \end{cases}$$

4.2 #52 The sum of two numbers is 956. One number is 94 less than 6 times the other. Find the numbers.

Aleks Problem:

Two systems of equations are given below. For each system, choose the best description of its solution. If applicable, give the solution.

$\begin{aligned}x + 5y &= 5 \\ -x - 5y &= 5\end{aligned}$	<p><input type="radio"/> The system has no solution</p> <p><input type="radio"/> The system has a unique solution: $(x, y) = (\square, \square)$</p> <p><input type="radio"/> The system has infinitely many solutions. They must satisfy the following equation: $y = \square$</p>
$\begin{aligned}x + 3y &= 3 \\ -x - 3y &= -3\end{aligned}$	<p><input type="radio"/> The system has no solution</p> <p><input type="radio"/> The system has a unique solution: $(x, y) = (\square, \square)$</p> <p><input type="radio"/> The system has infinitely many solutions. They must satisfy the following equation: $y = \square$</p>

4.2 #54 Two angles are complementary. One angle is 1° less than 6 times the other. Find the measure of each angle.

4.3 – Solving Systems using the Addition Method (aka Elimination Method)

1. Solve:
$$\begin{cases} 2x + y = 7 \\ 3x - y = 8 \end{cases}$$

2. Solve:
$$\begin{cases} 5x + 4y = 17 \\ -5x + 3y = 9 \end{cases}$$

3. Solve:
$$\begin{cases} x - 3y = 11 \\ 7x + 2y = 8 \end{cases}$$

4. Solve:
$$\begin{cases} 6c - 2d = -2 \\ 5c + 3d = 17 \end{cases}$$

5. Solve:
$$\begin{cases} 5m - 2n = 4 \\ 3m + n = 9 \end{cases}$$

6. Solve:
$$\begin{cases} -0.04x + 0.03y = 0.03 \\ -0.06x - 0.02y = -0.02 \end{cases}$$

4.3 #65 The difference of two positive numbers is 2. The sum of these numbers is 36. Find the numbers.

8. Solve:
$$\begin{cases} \frac{1}{4}x + \frac{2}{5}y = 6 \\ \frac{1}{2}x - \frac{1}{10}y = 3 \end{cases}$$

4.3 #67 Six times the smaller of 2 numbers minus the larger is -9 . Ten times the smaller number plus five times the larger number is 5. Find the numbers.

Miscellaneous Review

1. Solve:
$$\begin{cases} 8x - 2y = 5 \\ 12x + 4y = -3 \end{cases}$$

2. Solve:
$$\begin{cases} \frac{1}{3}x + \frac{1}{2}y = \frac{2}{3} \\ \frac{-2}{3}x + y = \frac{-4}{3} \end{cases}$$

4.4 – Word Problems

4.4 #6 The difference of two numbers is 264. The larger number is 3 times the smaller. Find the numbers.

4.4 #8 Two angles are supplementary. The measure of one angle is 9° more than twice the other. Find both angle measures.

4.4 #22 How many gallons of a 20% antifreeze and a 10% antifreeze solution must be mixed to obtain 40 gallons of a 16% antifreeze solution?

4.4 #24 How many gallons of a 25% antifreeze and a 15% antifreeze solution must be mixed to obtain 15 gallons of a 23% antifreeze solution?

4.4 #34 A kid is collecting state quarters and new \$1 coins. If she has 25 coins and the number of quarters is 9 more than the number of dollar coins, how many of each coin type does she have?

4.4 #16 \$12,000 was invested in two accounts, one earning 12% simple interest and the other earning 8%. If the total interest at the end of 1 year was \$1240, how much was invested at each rate?

Two cars start traveling toward each other from 600 miles apart. One travels 10 mph faster than the other. They meet in 4 hours. How fast is each driving?

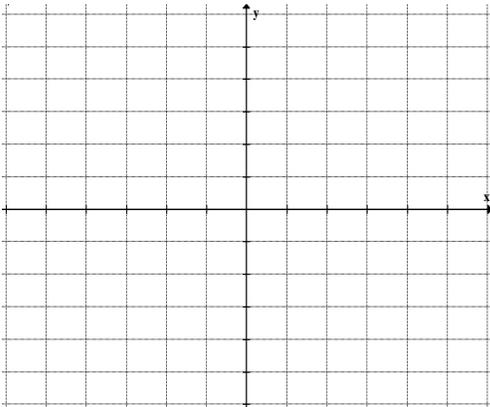
A hiker hikes uphill for 4 hours. Going downhill, the return trip takes 2 hours. The hiker's speed going down the hill is 2 mph faster than when going upward. How fast is the hiker going in each direction?

4.4 #28 A boat takes 1.5 hours to go 12 miles upstream against a current. It can go 24 miles downstream with the current in the same amount of time. Find the speed of the boat in still water and the speed of the current.

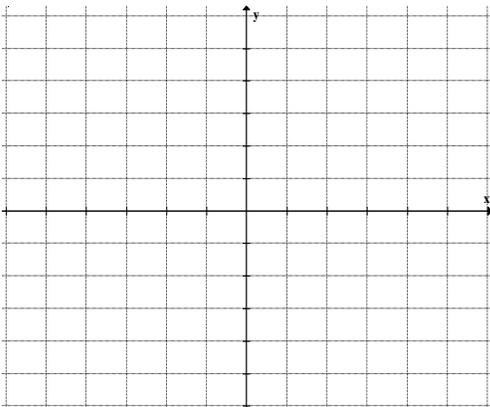
4.4 #30 A plane flies 720 miles with the wind in 3 hours. The return trip takes four hours. What is the wind speed, and what is the plane's speed in still air?

4.5 – Systems of Inequalities

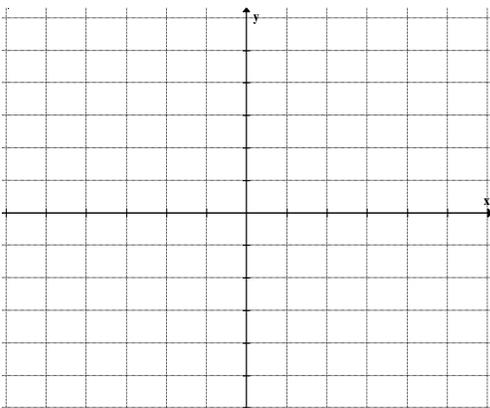
1. Graph: $y > \frac{2}{3}x - 1$



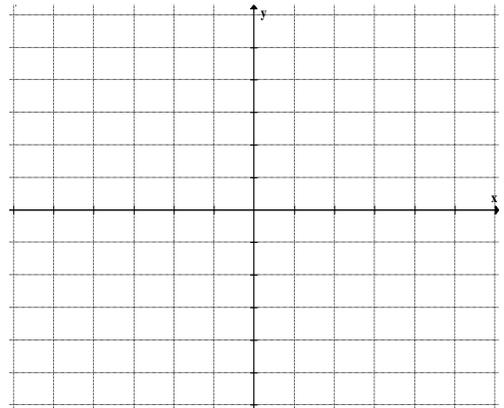
2. Graph: $y \leq \frac{-1}{3}x + 2$



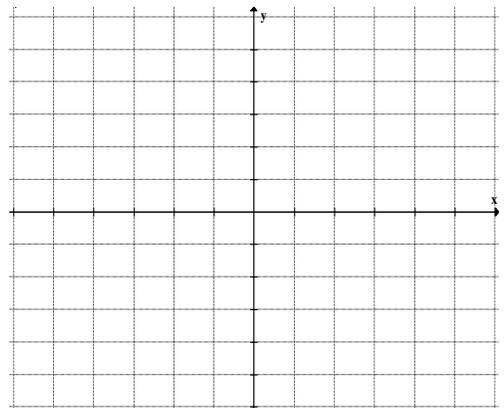
3. Graph: $-3x + 5y < 15$



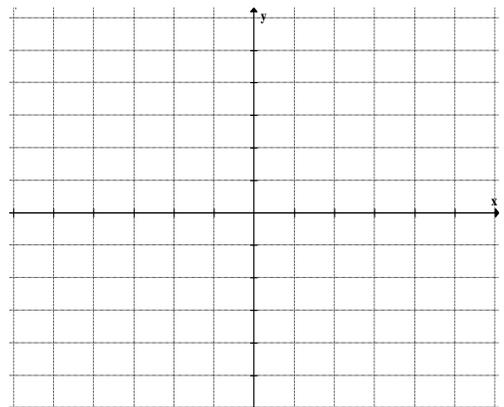
4. Graph: $x - 4y < 4$



5. Graph: $\begin{cases} y < \frac{-1}{2}x + 2 \\ y > x - 3 \end{cases}$



6. Graph: $\begin{cases} y > -x + 2 \\ y < 4 \\ x > 1 \end{cases}$

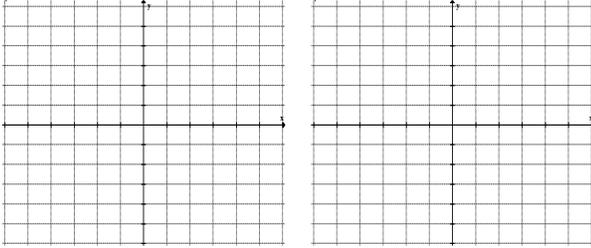


Review of Chapters 3 & 4

*Graph each of the following:

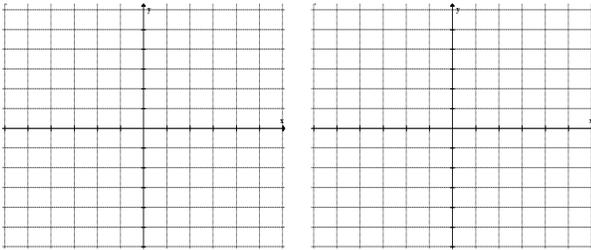
1. $2x - 3y = -6$

2. $y = \frac{1}{3}x - 4$



3. $x = 2$

4. $y = -x$



5. Find the slope between $(3, -4)$ and $(5, 9)$.

6. Find the equation of the line containing the points $(3, -4)$ and $(5, 9)$.

7. Find the slope of the lines which are parallel and perpendicular to $3x - 7y = 18$.

8. Find the equation of the line which passes through the point $(-2, 4)$ and is...

a. parallel to $3x - 7y = 18$.

b. perpendicular to $3x - 7y = 18$.

9. Find the x- and y-intercepts of $4x - 9y = 13$.

10. In 1995, 8 million homes were online. By the year 2005, 97 million homes were online. Let $x = 0$ represent the year 1995.

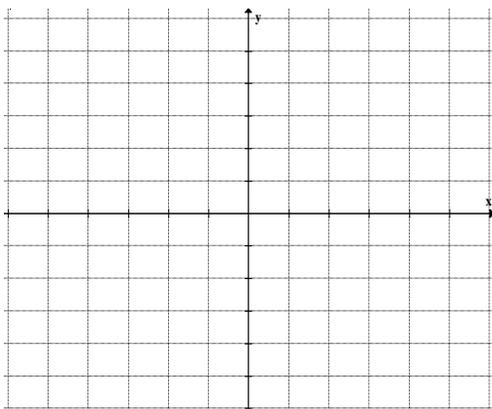
a. Find an equation to model this data.

b. What is the slope?

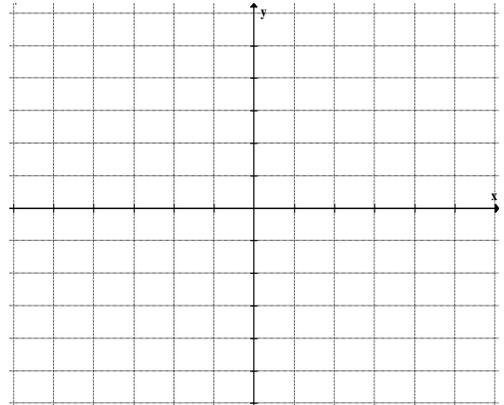
c. Interpret the slope in context.

d. Use the equation to predict the number of online homes in 2010.

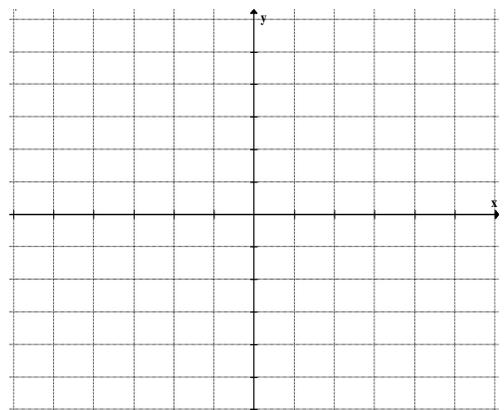
11. Solve by graphing: $\begin{cases} y = 3x - 2 \\ y = -x + 2 \end{cases}$



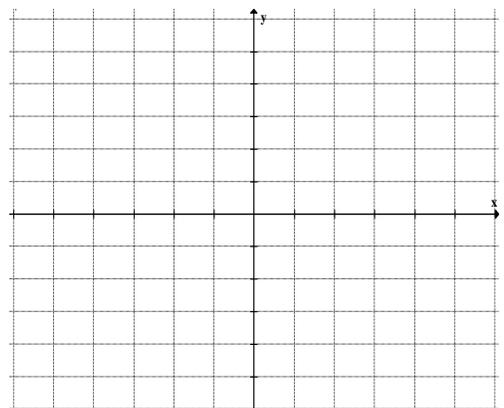
12. Graph: $y > -2x + 4$



13. Graph: $2x - 3y > 6$



14. Solve: $\begin{cases} x \geq -2 \\ y < \frac{1}{3}x - 1 \end{cases}$



15. Solve:
$$\begin{cases} x = y + 5 \\ 2x - 3y = 12 \end{cases}$$

16. Solve:
$$\begin{cases} -5x + 4y = -19 \\ 2x + 5y = 34 \end{cases}$$

17. Two angles are supplementary. The larger angle is 20° less than three times the smaller. Find both angle measures.

18. A 4% acid solution is mixed with a 20% acid solution. How many ml of each should be mixed in order to have 24 ml of a 10% acid?

19. A plane took 6 hours to fly to its destination with the wind and 8 hours on the return trip against the wind. If the distance of the trip is 3600 miles (each direction), find the speed of the plane and the wind speed.

20. Solve:
$$\begin{cases} 3x + 2y = 8 \\ y = \frac{-3}{2}x + 4 \end{cases}$$

21. Pat bought a combination of 42-cent stamps and 50-cent stamps. If she spends exactly \$22.60 on fifty stamps, how many of each did she buy?

22. Solve:
$$\begin{cases} \frac{1}{3}x + y = \frac{7}{3} \\ x = \frac{3}{2}y - 11 \end{cases}$$

23. Tina bought 3 adult tickets and 1 child's ticket to a movie for \$23. Lisa bought 2 adult tickets and 5 child tickets for \$30.50. Find the cost of each type of ticket.