

5.1, 5.2, 5.3 – Properties of Exponents

last revised 12/28/2010

Properties of Exponents

1. $x^a \cdot x^b = x^{a+b}$

2. $\frac{x^a}{x^b} = x^{a-b}$

3. $(x^a)^b = x^{a \cdot b}$

4. $x^0 = 1$

5. $x^{-a} = \frac{1}{x^a}$

*Simplify each of the following:

a. $x^4 \cdot x^8 =$

b. $x^5 \cdot x^7 \cdot x =$

c. $5^6 \cdot 5^{11} =$

d. $\frac{x^{14}}{x^9} =$

e. $\frac{x^6 y^{11} z^{14}}{x^3 y^7 z^{12}} =$

f. $(2x^2 y^{15,000})^0 =$

g. $3x^0 =$

h. $(3x)^0 =$

i. $\frac{x^{-4}}{x^2} =$

j. $\frac{x^3 y^{-4}}{x^{-2} y^8} =$



Negative exponents are NOT considered to be simplified. Do NOT leave them in final answers!



k. $\left(\frac{3}{5}\right)^2 =$

l. $(3x^2y^3)^4 =$

m. $(2x^2y^3)^2(-3xy^4)^2 =$

n. $\frac{x^{-4}}{x^{-8}} =$

o. $\frac{5^{-1}}{5} =$

p. $6^{-2} =$

q. $-6^{-2} =$

r. $\frac{-12x^{-4}y^{-3}}{48x^{-7}y^5} =$

s. $\left(\frac{3x^{-2}y^4}{6x^5y^{-7}}\right)^{-3} =$

6. **ALEKS PROBLEM TYPE:**

Simplify: $\left(\frac{-3u^{-3}}{w^{-6}}\right)(-2u^2v^3w^2)^{-3}$

Multiple choice: $(3x^3)^3 =$

- a. $9x^6$
- b. $9x^9$
- c. $27x^6$
- d. $27x^9$

5.4 – Scientific Notation

Scientific notation is a shorthand notation for writing extremely small or large numbers.

Notation:

*Write each using scientific notation:

1. 9,374,000

2. 19.4 trillion

3. 0.000381

*Write each in standard form:

4. 4.71×10^8

5. 3.21×10^{-5}

*Multiply. Write your answers in scientific notation:

6. $(3.5 \times 10^{11})(4.0 \times 10^{23})$

7. $(2.45 \times 10^{17})(3.5 \times 10^{12})$

*Divide. Write your answers in scientific notation:

8. $\frac{12.5 \times 10^{-4}}{2.5 \times 10^{19}}$

9. $\frac{2.4 \times 10^8}{4.8 \times 10^{42}}$

**Suggested Assignment: pg. 389 #1-40

5.5 – Adding and Subtracting Polynomials

monomial

binomial

trinomial

polynomial

Vocabulary: $ax^n + bx^{n-1} + \dots + cx + d$

*Given: $5x^7 + 4x^6 + 3x^5 \dots + 5x - 11$, find the following:

- leading coefficient
- constant term
- degree of the second term
- degree of the polynomial

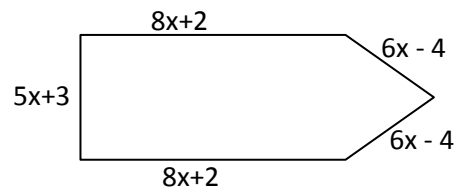
If a term has more than one variable, its degree is the _____ of its exponents.

*What is the degree of the expression $5x^2y^7$?

*Add: $(3x^2 + 5x - 2) + (7x^2 - 9x + 13)$

$$\begin{array}{r} 5x^3 + 3x^2 + 11 \\ *Add: \underline{8x^3 - 9x^2 + 5x - 3} \end{array}$$

*Find the perimeter:



$$*Subtract: (3x^2 + 5x + 11) - (x^2 + 7x - 4)$$

$$\begin{array}{r} 5x^3 - 2x^2 + 4x - 9 \\ *Subtract: \underline{2x^3 + 7x^2 - 11x + 8} \end{array}$$

5.6 – Multiplying Polynomials

*Multiply each of the following:

1. $(-3x^4)(5x^5)$

2. $4x(3x - 7)$

3. $5a^2b^3c^4(3ab^7 - 5ab^2d^5)$

4. $(x + 5)(x - 3)$

5. $(2x - 3)(3x + 5)$

6. $(3x - 5)(2x + 4)$

7. $(5x - 1)(x + 8)$

8. $(4x - 7)(4x + 7)$

9. $(a + b)(c + d)$

10. $(3x - 2)(x^2 + 4x - 7)$

11. $(5x + 7)(3x - 2)$

12. $(3x - 2)^2$

13. $(3x^2y^4)^2$

Mini-Review of Sections 5.1 – 5.6

1. Simplify: $\frac{-5x^{-3}y}{10x^4y^{-5}}$

2. Multiply: $(4.3 \times 10^8)(3.0 \times 10^{17})$

3. Divide: $\frac{0.6 \times 10^{-15}}{2.4 \times 10^{11}}$

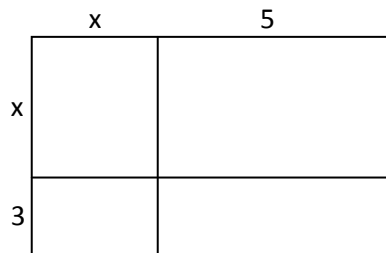
4. Multiply: $-5x^2y^3z(8xyz^5 - 4xy^2z^3)$

5. Multiply: $(3x^2 + 4)(2x^2 - 7)$

6. Multiply: $(x - 3)(x^2 + 3x + 9)$

7. Multiply: $(x^2 + 4x - 2)(x^2 + x - 5)$

8. Calculate the area of the larger rectangle in two ways.



5.7 – Dividing Polynomials

A. Dividing by a monomial

Create separate fractions and then simplify each separately.

$$1. \frac{10x^4y^3 + 15x^2y^8}{10x^2y^9}$$

$$2. (8x^3y - 4x^7y^5 + 2x^2y^4) \div (4xy^8)$$

B. Dividing by a non-Monomial

Use long division.

Recall... $512 \div 31$

$$3. (x^3 + 4x^2 - 2x + 8) \div (x - 1)$$

$$4. \frac{4t^3 + 4t^2 - 9t + 3}{2t + 3}$$

5. $(4x^3 + 5x - 12) \div (2x - 3)$

6. $(x^3 - 27) \div (x - 3)$

7. $(p^4 - p^3 - 4p^2 - 2p - 15) \div (p^2 + 2)$

In Math 90, you will learn another method for dividing called Synthetic Division. This process will only work when dividing by a linear factor (those without exponents). Problems like #7 above cannot be done using synthetic division since there is an exponent in the divisor.

***Suggested Assignment: pg. 389 #1-40*

6.1 – Introduction to Factoring

Factoring is _____ .

Factoring is a _____ process.

A. Factoring Out a Greatest Common Factor

*Factor each of the following completely.

1. $24x - 36$

2. $18x^2 - 18x$

3. $20x^5y^3z^2 - 24x^2y^5z$

4. $14x^5y^3 - 28x^7y^2 + 35x^2y^8$

5. $-12x^3 + 4x^2 - 9$

B. Factoring by Grouping

6. $x^2(x - 5) + 7(x - 5)$

7. $5x(x^3 + 2) - 8(x^3 + 2)$

8. $3q + 3p + qr + pr$

9. $8w^5 + 12w^2 - 10w^3 - 15$

10. $2c + 3ay + ac + 6y$

11. $12x^2 + 6x + 8x + 4$

12. $6f^2k + 30k + 2f^2 + 10$

Review

1. Multiply: $(x + 1)(x + 2)(x + 3)$

2. Simplify: $\frac{5^3}{5^{-8}}$

3. Simplify: $(5x^3y^2)^{-2}$

4. Simplify: $\left(\frac{4x^{-3}y}{6xy^{-5}}\right)^{-3}$

5. Multiply: $(5.6 \times 10^{14})(3.0 \times 10^{22})$

6. Multiply: $-4x^2y^3z(8xy^5 - 11x^2z^3)$

7. Multiply: $(5x^2y - 8z)^2$

8. Divide:

$$(16x^4y^2 + 20x^5y - 24x^3y^8) \div (6x^2y^7)$$

9. Divide: $(x^3 + 6x - 7) \div (x + 1)$

10. Divide: $(m^3 - 64) \div (m - 4)$

11. Factor: $-8x^3 + 16x^2 + 20x$

12. Factor: $5x^2(x + 7) - 8(x + 7)$

13. Factor: $2c + ac + 3ay + 6y$

14. Factor: $xy - xz + 7y - 7z$

15. Factor: $4x^3 + 3x^2y + 4xy^2 + 3y^3$

6.2 – Factoring Trinomials, part 1

* Factor each of the following:

1. $x^2 + 10x + 16$

2. $x^2 - 3x - 18$

3. $x^2 + 6x - 40$

4. $m^2 - 12m + 11$

5. $n^2 + 8n + 16$

6. $w^2 - 7w + 12$

7. $12p^2 - 96p + 84$

8. $x^3y^3 - 19x^2y^3 + 60xy^3$

9. $-2m^2 + 22m - 20$

10. $5w^2 - 40w - 45$

6.3 – Factoring Trinomials, part 2

pattern: $ax^2 + bx + c$

*Factor each of the following completely.

1. $3x^2 + 13x + 4$

2. $7y^2 + 9y - 10$

3. $8 + 7x^2 - 18x$

4. $12c^2 - 5c - 2$

5. $12y^2 - 73yz + 6z^2$

6. $36x^2 - 18x - 4$

7. $12m^2 + 11mn - 5n^2$

8. $y^2 - 6y - 40$

9. $16x^2 + 24x + 9$

10. $6p^4 + 17p^2 + 10$

11. $3y^3 - y^2 + 12y$

6.4 – Factoring Trinomials, part 3The a-c Method

1. $6x^2 + 7x - 3$

2. $9x^2 - 12x + 4$

3. $16x^2 + 10x + 1$

4. $16x^2 - 34x - 15$

6.5 – The Difference of Two Squares and Perfect Square Trinomials

The Difference of Two Squares

*Factor each completely:

1. $x^2 - 49$

2. $x^2 - 64$

3. $x^2 - 25$

4. $x^2 - 10$

5. $x^2 - \frac{1}{36}$

6. $x^2 + 25$

7. $x^2y^2 - 100z^2$

8. $x^4 - 16$

9. $x^8 - y^8$

10. $x^2 - 1$

11. $25x^2 - 16$

12. $100x^2 - 49y^2$

13. $25x^2 - 100$

14. $x^2 - 6xy + 9y^2 - 16$

15. $x^2 + 8x + 16 - 25y^2$

16. $w^2 - 10w + 25 - 36q^2$

17. $100 - x^2 - 16xy - 64y^2$

6.6 – The Sum & Difference of Two Cubes

Memorize:

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

"SOAP" means....

*Factor each completely:

1. $x^3 + 125$

2. $y^3 - 64$

3. $8x^3 - 1$

4. $3x^3 + 81$

What if you forget these formulas?

A General Strategy for Factoring

1. Can I factor out a _____ ?
2. How many terms are there?
 - a. if four, try _____.
 - b. If three, try _____.
 - c. If two, try _____
or try _____.
3. Can I factor further?

The following exercises are from pages 471-472 of your textbook, which are printed on the next page for your use.

#5. $2a^2 - 162$

#15. $3q^2 - 9q - 12$

#25. $64 + 16k + k^2$

#10. $5r^3 + 5$

#30. $3y^2 + y + 1$

#35. $-p^3 - 5p^2 - 4p$

#45. $14u^2 - 11uv + 2v^2$

#55. $81u^2 - 90uv + 25v^2$

#65. $12x^2 - 12x + 3$

5. $2a^2 - 162$
8. $16z^4 - 81$
11. $3ac + ad - 3bc - bd$
14. $7p^2 - 29p + 4$
17. $18a^2 + 12a$
20. $4t^2 - 31t - 8$
23. $x^3 + 0.001$
26. $s^2t + 5t + 6s^2 + 30$
29. $a^3 - c^3$
32. $a^2 + 2a + 1$
35. $-p^3 - 5p^2 - 4p$
38. $20y^2 - 14y + 2$
41. $t^2 + 2t - 63$
44. $6x^3y^4 + 3x^2y^5$
47. $4q^2 - 8q - 6$
50. $5b^2 - 30b + 45$
53. $16a^4 - 1$
56. $4x^2 + 16$
59. $2ax - 6ay + 4bx - 12by$
62. $2m^4 - 128$
65. $12x^2 - 12x + 3$
68. $4k^3 + 4k^2 - 3k$
71. $b^2 - 4b + 10$
6. $y^2 + 4y + 3$
-  9. $3t^2 + 13t + 4$
12. $x^3 - 125$
15. $3q^2 - 9q - 12$
18. $54 - 2y^3$
21. $10c^2 + 10c + 10$
24. $4q^2 - 9$
27. $2x^2 + 2x - xy - y$
30. $3y^2 + y + 1$
33. $b^2 + 10b + 25$
36. $x^2y^2 - 49$
39. $5a^2bc^3 - 7abc^2$
42. $b^2 + 2b - 80$
45. $14u^2 - 11uv + 2v^2$
48. $9w^2 + 3w - 15$
51. $6r^2 + 11r + 3$
54. $p^3 + p^2c - 9p - 9c$
57. $x^2 - 5x - 6$
60. $8m^3 - 10m^2 - 3m$
-  63. $8uv - 6u + 12v - 9$
66. $p^2 + 2pq + q^2$
69. $64 - y^2$
72. $y^2 + 6y + 8$
7. $6w^2 - 6w$
10. $5r^3 + 5$
13. $y^3 + 8$
16. $-2x^2 + 8x - 8$
-  19. $4t^2 - 100$
22. $2xw - 10x + 3yw - 15y$
-  25. $64 + 16k + k^2$
28. $w^3 + y^3$
31. $c^2 + 8c + 9$
34. $-t^2 - 4t + 32$
37. $6x^2 - 21x - 45$
40. $8a^2 - 50$
43. $ab + ay - b^2 - by$
46. $9p^2 - 36pq + 4q^2$
49. $9m^2 + 16n^2$
52. $4s^2 + 4s - 15$
55. $81u^2 - 90uv + 25v^2$
58. $q^2 + q - 7$
61. $21x^4y + 41x^3y + 10x^2y$
64. $4t^2 - 20t + st - 5s$
67. $6n^3 + 5n^2 - 4n$
70. $36b - b^3$
73. $c^4 - 12c^2 + 20$

6.7 – Solving Equations Using the Zero Product Rule

Quadratic equations are of the form

Zero Product Rule:

If $A \cdot B = 0$, Then $A = 0$ or $B = 0$.

Solve each of the following equations.

1. $(x + 2)(x - 3) = 0$

2. $(x + 5)(2x - 3) = 0$

3. $(8x - 5)(7x + 2) = 0$

4. $\frac{1}{3}z\left(z - \frac{5}{8}\right) = 0$

5. $x^2 - 2x - 15 = 0$

6. $x^2 - 8x + 16 = 0$

7. $x^2 - 24 = 2x$

8. $x^2 - 25 = 0$

9. $2x^2 - 50 = 0$

10. $x^3 - 25x = 0$

11. $2x^3 - 50x = 0$

$$6.7 \text{ \#26 } y^2 - 7y - 8 = 0$$

$$6.7 \text{ \#28 } w^2 - 10w + 16 = 0$$

$$6.7 \text{ \#30 } 4x^2 - 11x = 3$$

$$6.7 \text{ \#35 } 2m^3 - 5m^2 - 12m = 0$$

$$6.7 \text{ \#37 } 4(2x - 1)(x - 10)(x + 7) = 0$$

$$6.7 \text{ \#46 } 2y^2 - 20y = 0$$

$$6.7 \text{ \#48 } 9n^2 = 1$$

$$6.7 \text{ \#49 } 2y^3 + 14y^2 = -20y$$

$$6.7 \text{ \#62 } 3x(x - 2) - x = 3x^2 + 4$$

$$6.7 \text{ \#69 } (x - 1)(x + 2) = 18$$

6.8 – Quadratic Word Problems

A. Number Problems

6.8 #14 If a number is added to two times its square, the result is 36. Find all such numbers.

B. Consecutive Integer Problems

Review...

Consecutive Integers

Consecutive Even Integers

Consecutive Odd Integers

6.8 #14 The product of two even consecutive integers is 48. Find all such numbers.

6.8 #16 The sum of the squares of two consecutive integers is 9 less than 10 times their sum. Find all such integers.

The product of two consecutive odd integers is 63. Find all such integers.

The perimeter of a rectangle is 22 cm and its area is 24cm^2 . Find the length and width of this rectangle.

C. Area Problems

6.8 #18 The width of a rectangular picture frame is 2 inches less than its length. The area is 120 in^2 . Find its dimensions.

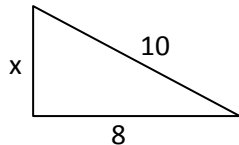
The length of a rectangle is three times its width. Find the dimensions if the area is 48 cm^2 .

D. Height of a Projectile

6.8 #24 A stone is dropped off a 256-ft. cliff. The height of the stone is given by $h = -16t^2 + 256$, where t is the time (in seconds). When will it hit the ground?

E. The Pythagorean Theorem

Find x:



6.8 #40 The longer leg of a right triangle is 1 cm less than twice the shorter leg. The hypotenuse is 1 cm more than twice the shorter leg. Find the length of the shorter leg.

Review of Chapters 5 & 6

1. Simplify: $(5x^2y)(-3x^4y^3)$

2. Simplify: $5x^0$

3. Simplify: 3^{-3}

4. Simplify: $(x^5)^3$

5. Simplify: $\left(\frac{3x^{-2}y^4}{6x^5y^{-2}}\right)^{-3}$

6. Simplify: $\left(\frac{-4x^{-5}y^{-3}z}{8x^{-7}y^4z^{-3}}\right)^{-4}$

7. Write 5.37×10^4 in standard form.

8. Write 365,000,000 in scientific notation.

9. Multiply: $(3.5 \times 10^{11})(4.0 \times 10^{23})$

10. Divide: $\frac{6.0 \times 10^4}{8.0 \times 10^{23}}$

11. Divide: $\frac{x^3+64}{x+4}$

12. Divide:

$$(5x^3 + 10x^2 - 15x + 20) \div (15x^3)$$

13. Simplify: $(3x - 2y)^2$

14. Add: $(4x + 2) + (3x - 1)$

15. Subtract $3x^2 - 4x + 8$ from $x^2 - 9x - 11$.

16. Multiply: $(3x + 5)(2x - 7)$

17. Multiply: $(x - 4)(x^2 + 5x - 3)$

18. The square of a number is subtracted from 60, resulting in -4 . Find all such numbers.

19. The product of consecutive integers is 44 more than 14 times their sum. Find all such integers.

20. The length of a rectangle is 1 ft. longer than twice its width. If the area is 78 ft^2 , find the rectangle's dimensions.

21. A right triangle has one leg that is 2 ft. longer than the other leg. The hypotenuse is 2 ft. less than twice the shorter leg. Find the lengths of all three sides of the triangle.

22. Factor: $x^2 + x - 42$

23. Factor: $c^4 - 1$

24. Factor: $-10u^2 + 30u - 20$

25. Factor: $y^3 - 125$

26. Factor: $49 + p^2$

27. Factor: $2x^3 + x^2 - 8x - 4$

28. Factor: $3a^2 + 27ab + 54b^2$

ALEKS TOPIC - Domain of a rational function

The function h is defined below.

$$h(x) = \frac{x^2 - 6x - 27}{x^2 - 81}$$

Find all values of x that are NOT in the domain of h . If there is more than one value, separate them with commas.

same directions....

$$f(x) = \frac{x + 7}{x + 8}$$

$$g(x) = \frac{18}{x(x-4)}$$

Additional Review (Chapters 5&6) – If Time

1. Write 463,000,000 in scientific notation.

2. Multiply: $(2.5 \times 10^{17})(6.0 \times 10^{-4})$

3. Divide: $\frac{6.0 \times 10^{-4}}{8.0 \times 10^{19}}$

4. Simplify each:

a. $5x^0$

b. $(5x)^0$

c. $\left(\frac{-2}{3}\right)^2$

d. -8^{-2}

e. $(-8)^{-2}$

f. $(-3m^2n^3)^4$

g. $\frac{-5x^{-2}y^3}{-10x^4y^{-5}}$

h. $\left(\frac{-3u^{-2}v}{w^{-3}}\right)^{-2} \cdot (uv^{-2})$

5. Given $5x^2 + 3x^7 - 2x + 8$, state find each:

a. Leading coefficient: _____

b. degree of the polynomial: _____

6. Find the degree of the polynomial:

$$x^2y^4z + 3xy^7z^2 - 2x^4yz^3$$

7. Add: $(2x - 5) + (4x + 8)$

8. Combine:

$$(x^2 + 4x - 9) - (2x^2 - 3x + 4) + (5x^2 - 11)$$

9. Multiply each:

a. $(3x - 5)(2x + 7)$

b. $(4x + 7)(4x - 7)$

c. $(3x + 2y)(4x - 9y)$

d. $(5x - 8)^2$

e. $(x + 5)(x^2 - x + 2)$

10. Divide: $\frac{5x^2y^3 - 12xy^4 + 10x^4y}{8x^3y^2}$

11. Divide: $(6x^2 - 5x + 4) \div (x - 3)$

12. Factor: $5x^2 + 10x + 100$

13. Factor: $60xa - 30xb - 80ya + 40yb$

14. Factor: $x^2 - 3x - 28$

15. Solve: $x^2 - 3x - 28 = 0$

16. Factor: $2x^2 - x - 6$

17. Factor: $x^4 - 16$

18. Factor: $x^3 - 27y^3$

19. Solve: $x^2 = 8x$

20. The length of a rectangle is 2 cm more than three times its width. Find the dimensions if the area is 56 cm^2 .