

# Lecture Guide

## Math 90 - Intermediate Algebra

to accompany

Intermediate Algebra, 3rd edition

Miller, O'Neill, & Hyde

Prepared by

**Stephen Toner**

Victor Valley College

Last updated: 7/10/14

## 4.1 – Solving Systems by Graphing

In consistent systems,

Independent systems consist of

### Three Cases:

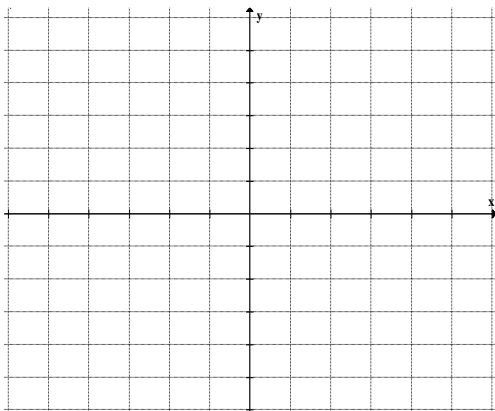
A. consistent and independent

B. inconsistent and independent

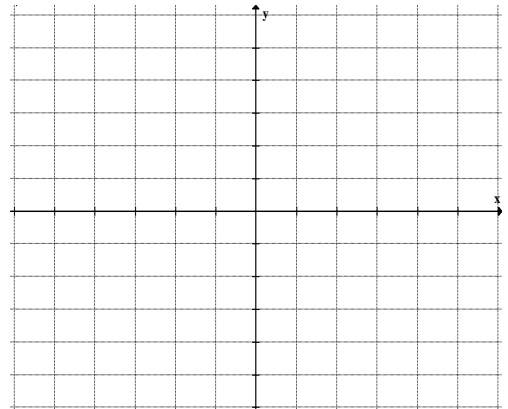
C. consistent and dependent

\*Find the solution to each system by graphing.

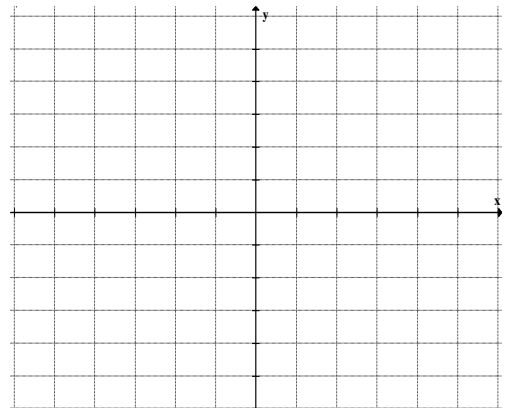
1. 
$$\begin{cases} y = -2x \\ y = -3x + 1 \end{cases}$$



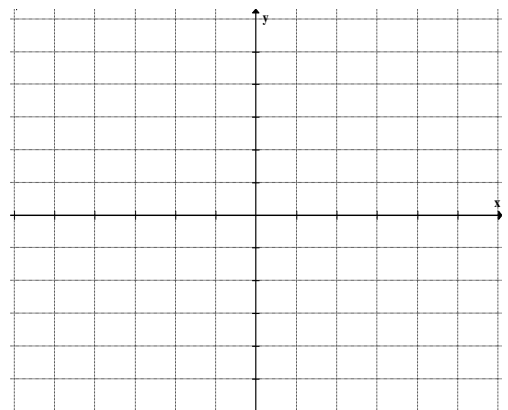
2. 
$$\begin{cases} y = \frac{1}{2}x + 4 \\ x - 2y = -4 \end{cases}$$



3. 
$$\begin{cases} y = -3x + 4 \\ y = 2x - 1 \end{cases}$$



4. 
$$\begin{cases} 2x + y = 4 \\ 4x - 2y = -4 \end{cases}$$



#### 4.2 – Solving Systems by Substitution

1. Solve: 
$$\begin{cases} 10y + 34 = x \\ -7x + y = -31 \end{cases}$$

2. Solve: 
$$\begin{cases} 5x - 2y = 10 \\ y = x - 1 \end{cases}$$

3. Solve: 
$$\begin{cases} x + 4y = 8 \\ 3x = 3 - 12y \end{cases}$$

#### 4.3 – Solving Systems by the Addition Method

1. Solve: 
$$\begin{cases} 2x + y = 7 \\ 3x - y = 8 \end{cases}$$

2. Solve: 
$$\begin{cases} x - 3y = -14 \\ 7x + 2y = -24 \end{cases}$$

3. Solve: 
$$\begin{cases} 6c - 2d = -2 \\ 5c + 3d = 17 \end{cases}$$

4. Solve: 
$$\begin{cases} 2(2y + 3) - 2x = 1 - x \\ x + y = \frac{1}{5}(7 + y) \end{cases}$$

6. Solve: 
$$\begin{cases} 4x - 2y = 6 \\ x = \frac{1}{2}y + \frac{3}{2} \end{cases}$$

5. Solve: 
$$\begin{cases} \frac{1}{10}x - \frac{1}{2}y = -\frac{8}{5} \\ x + \frac{1}{4}y = -\frac{11}{2} \end{cases}$$

7. Find all values of  $k$  such that the system of equations 
$$\begin{cases} kx - 12y = -10 \\ -15x + 10y = -19 \end{cases}$$

does not have a solution.

#### 4.4 – Word Problems

##### A. Cost

1. John and Ariana bought school supplies. John spent \$10.65 on 4 notebooks and 5 pens. Ariana spent \$7.50 on 3 notebooks and 3 pens. What is the cost of 1 notebook and what is the cost of 1 pen?

##### B. Mixture

2. How much 30% acid solution should be added to 10% acid solution to make 100 mL of a 12% acid solution?

3. A fruit punch that contains 25% fruit juice is combined with 100% fruit juice. How many ounces of each should be used to make 48 oz of a mixture that is 75% fruit juice?

C. Money

4. Aliya deposited half as much money in a savings account earning 2.5% simple interest as she invested in a money market account that earns 3.5% simple interest. If the total interest after one year is \$247, how much did she invest in each account?

5. Jody invested \$5000 less in an account paying 4% simple interest than she did in an account paying 3% simple interest. At the end of the first year, the total interest from both accounts was \$675. Find the amount invested in each account.

D. Motion

6. A plane flies from Atlanta to Los Angeles against the wind in 5 hr. The return trip back to Atlanta with the wind takes only 4 hr. If the distance between Atlanta and Los Angeles is 3200 km, find the speed of the plane in still air and the speed of the wind.

7. Kim rides a total of 48 km in the bicycle portion of a triathlon. The course is an “out and back” route. It takes her 3 hr on the way out against the wind. The ride back takes her 2 hr with the wind. Find the speed of the wind and Kim’s speed riding her bike in still air.

E. Geometry

8. Two angles are supplementary. One angle measures  $2^\circ$  less than 3 times the other. What are the measures of the two angles?

9. Two angles are complementary. One angle measures  $15^\circ$  more than 2 times the measure of the other. What are the measures of the two angles?

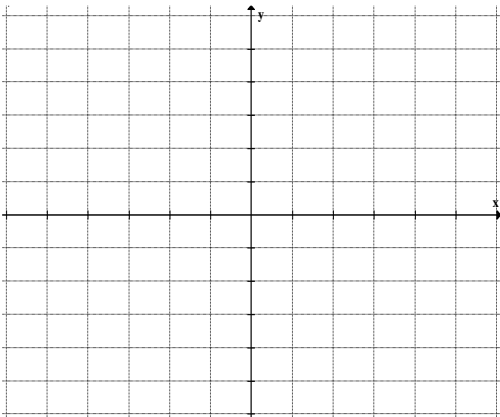
10. The difference of two positive numbers is 2. The sum of these numbers is 36. Find the numbers.

11. Six times the smaller of 2 numbers minus the larger is  $-9$ . Ten times the smaller number plus five times the larger number is 5. Find the numbers.

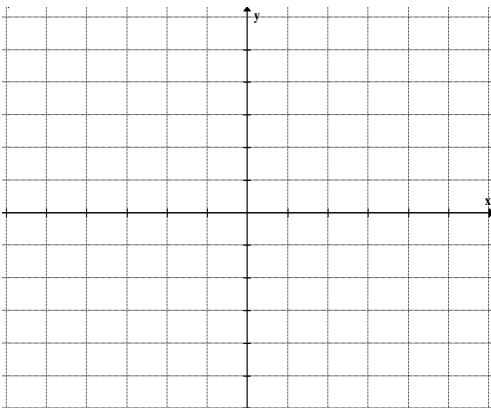
## 4.5 – Systems of Inequalities

Rules for Shading:

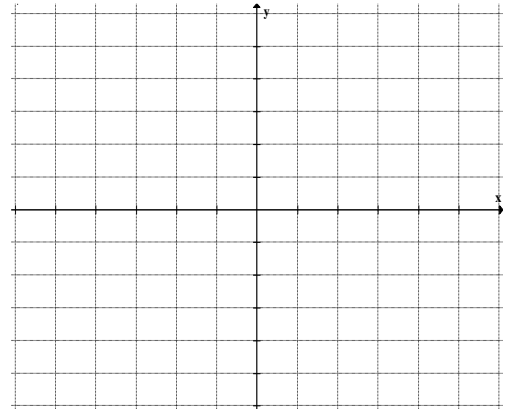
$$\text{Solve: } \begin{cases} y > -2x + 3 \\ y \leq \frac{1}{2}x - 2 \end{cases}$$



$$\text{Solve: } \begin{cases} x < 2 \\ y \geq 1 \end{cases}$$



Graph:  $5x - 3y < 15$



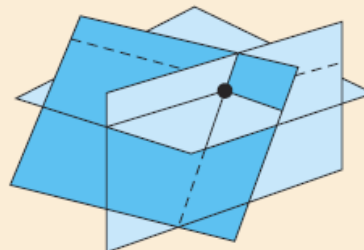
On the final exam, Ahmad needs more than 60 points to get an A in the class. The exam has true/false questions, worth 2 points each, and multiple choice questions, worth 4 points each. Let  $x$  be the number of true/false questions he gets correct, and let  $y$  be the number of multiple choice questions he gets correct. Write an inequality showing how Ahmad can get enough points for an A in the class.



## 4.6 – Systems in Three Variables

One unique solution (planes intersect at one point)

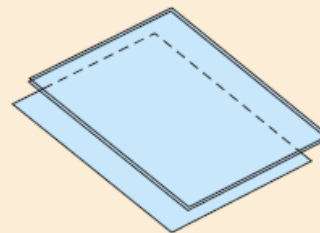
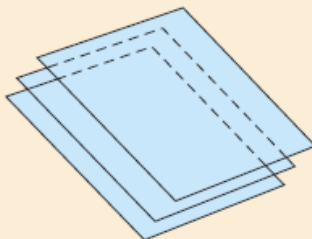
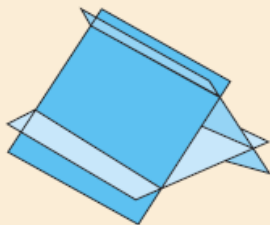
- The system is consistent.
- The system is independent.



**Table 3-3**

No solution (the three planes do not all intersect)

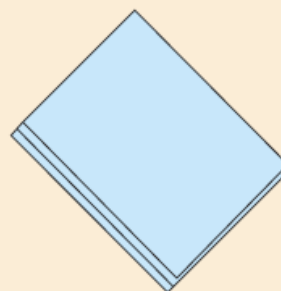
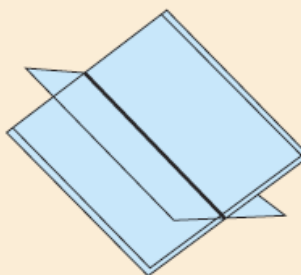
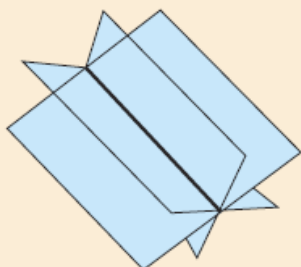
- The system is inconsistent.
- The system is independent.



**Table 3-4**

Infinitely many solutions (planes intersect at infinitely many points)

- The system is consistent.
- The system is dependent.



$$\text{Solve: } \begin{cases} x - 3y - 4z = -7 \\ 5x + 2y + 2z = -1 \\ 4x - y - 5z = -6 \end{cases}$$

$$\text{Solve: } \begin{cases} x + y + z = 6 \\ -x + y - z = -2 \\ 2x + 3y + z = 11 \end{cases}$$

$$\text{Solve: } \begin{cases} 3x + 2z = 11 \\ y - 7z = 4 \\ x - 6y = 1 \end{cases}$$

The largest angle of a triangle measures  $4^\circ$  less than 5 times the measure of the smallest angle. The middle angle measures twice that of the smallest angle. Find the measures of the three angles.

The perimeter of a triangle is 5 ft. The longest side of the triangle measures 20 in. more than the shortest side. The middle side is 3 times the measure of the shortest side. Find the lengths of the three sides *in inches*.

## A2 – Determinants and Cramer's Rule

### A. Determinants

Evaluate:  $\begin{vmatrix} 6 & 2 \\ 3 & -7 \end{vmatrix}$

Evaluate:  $\begin{vmatrix} 6 & 8 \\ -4 & 7 \end{vmatrix}$

In general,  $\begin{vmatrix} a & c \\ b & d \end{vmatrix}$

Evaluate:  $\begin{vmatrix} 4 & 5 & 8 \\ 2 & -2 & 0 \\ -1 & 3 & 5 \end{vmatrix}$

Evaluate:  $\begin{vmatrix} 3 & 4 & -3 \\ 1 & 5 & 2 \\ -1 & -2 & 0 \end{vmatrix}$

B. Using Determinants

Use Cramer's Rule to solve the following systems:

$$\begin{cases} 4x + y = -11 \\ 3x - 5y = 60 \end{cases}$$

$$\text{Solve: } \begin{cases} x + 3y + 5z = 6 \\ 2x - 4y + 6z = 14 \\ 9x - 6y + 3z = 3 \end{cases}$$

#### 4.7 – Solving Systems Using Row Operations on Matrices

Write this system as an augmented matrix:

$$\begin{cases} 5x + 3y - 2z = 9 \\ x + 8y - 4z = 7 \\ -3x - y + 9z = 2 \end{cases}$$

Allowable Row Operations:

1. You may \_\_\_\_\_ rows.
2. You may \_\_\_\_\_ a row by a number (scalar).
3. You may add a \_\_\_\_\_ of one row to another row.

Goal/Process:

$$\text{Solve: } \begin{cases} 2x - 4y = 10 \\ 3x + 4y = 5 \end{cases}$$

$$\text{Solve: } \begin{cases} 3x + y + z = -2 \\ x - 2y + 3z = 1 \\ 2x - 3y + 5z = 3 \end{cases}$$

$$\text{Solve: } \begin{cases} 3x - 2y = 4 \\ 9x + 4y = -3 \end{cases}$$

### Some Chapter 4 Review Problems

1. Evaluate:  $\begin{vmatrix} 3 & -8 \\ 1 & 4 \end{vmatrix}$

2. Evaluate:  $\begin{vmatrix} -4 & 1 & -3 \\ 5 & 6 & 8 \\ -1 & 2 & 0 \end{vmatrix}$

3. Solve:  $\begin{cases} 5x - 2y = 10 \\ y = x + 1 \end{cases}$

4. Solve:  $\begin{cases} 4x - 5y = 19 \\ 2x + 3y = 15 \end{cases}$

5. Solve:  $\begin{cases} \frac{1}{3}x + y = \frac{7}{3} \\ x = \frac{3}{2}y - 11 \end{cases}$

6. Pat bought a combination of 42-cent stamps and 50-cent stamps. If she spends exactly \$22.60 on fifty stamps, how many of each did she buy?

7. A plane took 6 hours to fly to its destination with the wind and 8 hours on the return trip against the wind. If the distance of the trip is 3600 miles (each direction), find the speed of the plane and the wind speed.

8. Use row operations to solve:

$$\begin{cases} x - 3y = -1 \\ 4x + 5y = 30 \end{cases}$$

9. Solve: 
$$\begin{cases} x + y + z = 4 \\ -x + y - z = 6 \\ 2x + 3y + z = 16 \end{cases}$$

10. Use Cramer's Rule to solve the following:

$$\begin{cases} -2x + y = 10 \\ 3x - 5y = -290 \end{cases}$$