

Lecture Guide

Math 90 - Intermediate Algebra

to accompany

Intermediate Algebra, 3rd edition

Miller, O'Neill, & Hyde

Prepared by

Stephen Toner

Victor Valley College

Last updated: 4/17/16

5.1 – Exponents & Scientific Notation

A. Properties of Exponents

1. $x^a \cdot x^b = x^{a+b}$

2. $\frac{x^a}{x^b} = x^{a-b}$

3. $(x^a)^b = x^{a \cdot b}$

4. $x^0 = 1$

5. $x^{-a} = \frac{1}{x^a}$

*Simplify each of the following:

a. $x^4 \cdot x^8 =$

b. $x^5 \cdot x^7 \cdot x =$

c. $5^6 \cdot 5^{11} =$

d. $\frac{x^{14}}{x^9} =$

e. $\frac{x^6 y^{11} z^{14}}{x^3 y^7 z^{12}} =$



f. $(2x^2 y^{15,000})^0 =$

g. $3x^0 =$

h. $(3x)^0 =$

i. $\frac{x^{-4}}{x^2} =$

j. $\frac{x^3 y^{-4}}{x^{-2} y^8} =$

 Negative exponents are NOT considered to be simplified. Do NOT leave them in final answers! 

$$\text{k. } \left(\frac{3}{5}\right)^2 =$$

$$\text{l. } (3x^2y^3)^4 =$$

$$\text{m. } (2x^2y^3)^2(-3xy^4)^2 =$$

$$\text{n. } \frac{x^{-4}}{x^{-8}} =$$

$$\text{o. } \frac{5^{-1}}{5} =$$

$$\text{p. } 6^{-2} =$$

$$\text{q. } -6^{-2} =$$

$$\text{r. } \frac{-12x^{-4}y^{-3}}{48x^{-7}y^5} =$$

$$\text{s. Simplify: } \left(\frac{3x^{-2}y^4}{6x^5y^{-7}}\right)^{-3} =$$

$$\text{t. Simplify: } \left(\frac{-3u^{-3}}{w^{-6}}\right)(-2u^2v^3w^2)^{-3}$$

B. Scientific Notation

Scientific notation is a shorthand notation for writing extremely small or large numbers.

Notation:

*Write each using scientific notation:

1. 9,374,000

2. 19.4 trillion

3. 0.000381

*Write each in standard form:

4. 4.71×10^8

5. 3.21×10^{-5}

*Multiply. Write your answers in scientific notation:

6. $(3.5 \times 10^{11})(4.0 \times 10^{23})$

7. $(2.45 \times 10^{17})(3.5 \times 10^{12})$

*Divide. Write your answers in scientific notation:

8. $\frac{4.5 \times 10^{-4}}{1.5 \times 10^{19}}$

9. $\frac{2.4 \times 10^8}{4.8 \times 10^{42}}$

5.2 – Adding and Subtracting Polynomials

monomial

binomial

trinomial

polynomial

Vocabulary: $ax^n + bx^{n-1} + \dots + cx + d$

*Given: $5x^7 + 4x^6 + 3x^5 \dots + 5x - 11$, find the following:

- leading coefficient
- constant term
- degree of the second term
- degree of the polynomial

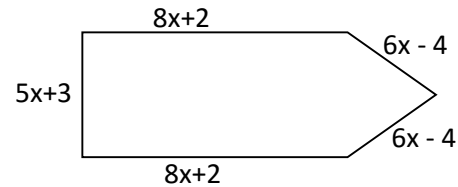
If a term has more than one variable, its degree is the _____ of its exponents.

*What is the degree of the expression $5x^2y^7$?

*Add: $(3x^2 + 5x - 2) + (7x^2 - 9x + 13)$

$$\begin{array}{r} 5x^3 + 3x^2 \quad + 11 \\ *Add: \underline{8x^3 - 9x^2 + 5x - 3} \end{array}$$

*Find the perimeter:



$$*Subtract: (3x^2 + 5x + 11) - (x^2 + 7x - 4)$$

$$\begin{array}{r} 5x^3 - 2x^2 + 4x - 9 \\ *Subtract: \underline{2x^3 + 7x^2 - 11x + 8} \end{array}$$

5.3 – Multiplying Polynomials

*Multiply each of the following:

1. $(-3x^4)(5x^5)$

2. $4x(3x - 7)$

3. $5a^2b^3c^4(3ab^7 - 5ab^2d^5)$

4. $(x + 5)(x - 3)$

5. $(2x - 3)(3x + 5)$

6. $(3x - 5)(2x + 4)$

7. $(5x - 1)(x + 8)$

8. $(4x - 7)(4x + 7)$

9. $(a + b)(c + d)$

10. $(3x - 2)(x^2 + 4x - 7)$

11. $(5x + 7)(3x - 2)$

12. $(3x - 2)^2$

13. $(3x^2y^4)^2$

5.4 – Dividing Polynomials

A. Dividing by a monomial

Create separate fractions and then simplify each separately.

1. $\frac{10x^4y^3 + 15x^2y^8}{10x^2y^9}$

2. $(8x^3y - 4x^7y^5 + 2x^2y^4) \div (4xy^8)$

B. Dividing by a non-Monomial

Use long division.

Recall... $512 \div 31$

3. $(x^3 + 4x^2 - 2x + 8) \div (x - 1)$

4. $\frac{4t^3 + 4t^2 - 9t + 3}{2t + 3}$

5. $(x^3 - 27) \div (x - 3)$

Long division **always** works; synthetic division only works when dividing by _____ factors (those without exponents).

*Divide using synthetic division:

$$(x^3 + 2x^2 + 4x - 5) \div (x + 1)$$

6. $(p^4 - p^3 - 4p^2 - 2p - 15) \div (p^2 + 2)$

steps:

Divide: $(x^3 + 4x^2 - 2x + 8) \div (x - 1)$

Use synthetic division to determine whether $x + 4$ is a factor of $f(x) = x^3 + 2x^2 - 5x - 6$.

Divide: $(x^3 + 64) \div (x + 4)$

Synthetic division can also be used to evaluate polynomials:

If $f(x) = x^4 - 3x^3 + 5x^2 - 8x + 17$, find $f(-2)$ in two ways.

5.5 – Factoring (GCF and Grouping)

A. Factoring Out a Greatest Common Factor

*Factor each of the following completely.

1. $24x - 36$

2. $18x^2 - 18x$

3. $20x^5y^3z^2 - 24x^2y^5z$

4. $14x^5y^3 - 28x^7y^2 + 35x^2y^8$

5. $-12x^3 + 4x^2 - 9$

B. Factoring by Grouping

6. $x^2(x - 5) + 7(x - 5)$

7. $5x(x^3 + 2) - 8(x^3 + 2)$

8. $3q + 3p + qr + pr$

9. $8w^5 + 12w^2 - 10w^3 - 15$

10. $2c + 3ay + ac + 6y$

11. $12x^2 + 6x + 8x + 4$

12. $6f^2k + 30k + 2f^2 + 10$

5.6 – Factoring Trinomials

* Factor each of the following:

1. $x^2 + 10x + 16$

2. $x^2 - 3x - 18$

3. $x^2 + 6x - 40$

4. $m^2 - 12m + 11$

5. $n^2 + 8n + 16$

6. $7y^2 + 9y - 10$

7. $8 + 7x^2 - 18x$

8. $12c^2 - 5c - 2$

9. $12y^2 - 73yz + 6z^2$

10. $36x^2 - 18x - 4$

11. $12m^2 + 11mn - 5n^2$

12. $16x^2 + 24x + 9$

13. $6p^4 + 17p^2 + 10$

14. $3y^3 - y^2 + 12y$

5.7 – Factoring - Special Cases

The Difference of Two Squares

*Factor each completely:

1. $x^2 - 49$

2. $x^2 - 64$

3. $x^2 - 25$

4. $x^2 - 10$

5. $x^2 - \frac{1}{36}$

6. $x^2 + 25$

7. $x^2y^2 - 100z^2$

8. $x^4 - 16$

9. $x^8 - y^8$

10. $x^2 - 1$

11. $25x^2 - 16$

12. $100x^2 - 49y^2$

13. $25x^2 - 100$

14. $x^2 - 6xy + 9y^2 - 16$

The Sum & Difference of Two Cubes

Memorize:

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

"SOAP" means....

*Factor each completely:

1. $x^3 + 125$

2. $y^3 - 64$

3. $8x^3 - 1$

4. $3x^3 + 81$

Factoring – General Strategy

1. Can I factor out a _____ ?
2. How many terms are there?
 - a. if four, try _____.
 - b. If three, try _____.
 - c. If two, try _____
or try _____.
3. Can I factor further?

*Factor each of the following completely.

1. $2a^2 - 162$

2. $3q^2 - 9q - 12$

3. $64 + 16k + k^2$

4. $5r^3 + 5$

5. $3y^2 + y + 1$

6. $-p^3 - 5p^2 - 4p$

7. $14u^2 - 11uv + 2v^2$

8. $81u^2 - 90uv + 25v^2$

9. $12x^2 - 12x + 3$

10. $t^4 - 8t$

5.8 – Quadratic Equations and Word Problems

Quadratic equations are of the form

Zero Product Rule:

If $A \cdot B = 0$, Then $A = 0$ or $B = 0$.

Solve each of the following equations.

1. $(x + 2)(x - 3) = 0$

2. $(x + 5)(2x - 3) = 0$

3. $x^2 - 2x - 15 = 0$

4. $x^2 - 8x + 16 = 0$

5. $x^2 - 24 = 2x$

$$6. x^2 - 25 = 0$$

$$7. 2x^2 - 50 = 0$$

$$8. x^3 - 25x = 0$$

$$9. 2x^3 - 50x = 0$$

$$10. 4x^2 - 11x = 3$$

$$11. 2m^3 - 5m^2 - 12m = 0$$

$$12. 2y^2 - 20y = 0$$

$$13. 2y^3 + 14y^2 = -20y$$

$$14. 3x(x - 2) - x = 3x^2 + 4$$

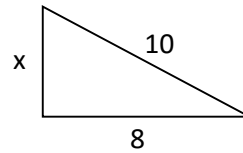
$$15. (x - 1)(x + 2) = 18$$

16. If a number is added to two times its square, the result is 36. Find all such numbers.

17. The length of a rectangle is three times its width. Find the dimensions if the area is 48 cm^2 .

18. A stone is dropped off a 256-ft. cliff. The height of the stone is given by $h = -16t^2 + 256$, where t is the time (in seconds). When will it hit the ground?

19. Use the Pythagorean Theorem to find x :



20. The longer leg of a right triangle is 1 cm less than twice the shorter leg. The hypotenuse is 1 cm more than twice the shorter leg. Find the length of the shorter leg.

21. Write the quadratic equation whose roots are -1 and 4 , and whose leading coefficient is 3 .

22. A 17-foot ladder is leaning against a wall. The distance between the base of the ladder and the wall is 7 feet less than the distance between the top of the ladder and the base of the wall. Find the distance between the base of the ladder and the wall.

23. Find the x- and y- intercepts of the function $f(x) = (x+1)(x-2)(x-3)^2$.

Chapter 5 Review

1. Divide: $\frac{x^3+64}{x+4}$

2. Divide:

$$(5x^3 + 10x^2 - 15x + 20) \div (15x^3)$$

3. Simplify: $(3x - 2y)^2$

4. Add: $(4x + 2) + (3x - 1)$

5. Subtract $3x^2 - 4x + 8$ from $x^2 - 9x - 11$.

6. Multiply: $(3x + 5)(2x - 7)$

7. Multiply: $(x - 4)(x^2 + 5x - 3)$

8. The square of a number is subtracted from 60, resulting in -4 . Find all such numbers.

9. The length of a rectangle is 1 ft. longer than twice its width. If the area is 78 ft^2 , find the rectangle's dimensions.

10. Factor: $x^2 + x - 42$

11. Factor: $c^4 - 1$

12. Factor: $-10u^2 + 30u - 20$

13. Factor: $y^3 - 27$

14. Factor: $49 + p^2$

15. Factor: $2x^3 + x^2 - 8x - 4$

16. Factor: $3a^2 + 27ab + 54b^2$

17. Solve $(x - 2)(x + 5) = 44$