

Lecture Guide

Math 90 - Intermediate Algebra

to accompany

Intermediate Algebra, 3rd edition

Miller, O'Neill, & Hyde

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Last updated: 7/21/14

7.1 – Definition of an nth Root

$\sqrt{25} =$ _____ since _____.

Question: Can $\sqrt{25} = -5$ since $(-5)^2 = 25$?

Answer:

Evaluate: $\sqrt{-25}$

Evaluate: $-\sqrt{25}$

Evaluate: $\sqrt[3]{-8}$

Evaluate: $\sqrt{(-8)^2}$

Property:

*Simplify each of the following:

1. $\sqrt{100}$

2. $\sqrt{144x^2}$

3. $\sqrt{225w^4}$

4. $\sqrt{169w^6}$

5. $\sqrt[4]{x^4}$

When, exactly, do you use absolute values?

6. $\sqrt[5]{32}$

7. $\sqrt[3]{27}$

8. $\sqrt[3]{-27}$

9. $\sqrt[5]{a^5}$

10. $\sqrt[5]{32x^5y^{10}}$

*Find the domain. Write it using interval notation.

11. $f(x) = \sqrt{x - 3}$

12. $g(x) = \sqrt[4]{5 - 3x}$

13. $h(x) = \sqrt[3]{2x - 5}$

7.2 – Rational Exponents

Definition: $x^{p/r} = \sqrt[r]{x^p} = (\sqrt[r]{x})^p$

1. Evaluate: $(27)^{4/3}$

2. Evaluate: $(4)^{5/2}$

3a. Evaluate: $81^{3/4}$

3b. Evaluate: $-81^{3/4}$

3c. Evaluate: $(-81)^{3/4}$

3d. Evaluate: $81^{-3/4}$

3e. Evaluate: $-81^{-3/4}$

3f. Evaluate: $(-81)^{-3/4}$

*Re-write each as a simplified radical:

4. $t^{3/5}$

5. $8b^{4/9}$

6. $(c^2d)^{1/6}$

7. $(7x)^{-1/4}$

*Re-write each using rational exponents:

8. $\sqrt[4]{a}$

9. $\sqrt[6]{z^5}$

10. $\sqrt[8]{a^2b^4c^7}$

*Use properties of exponents to simplify each.
(Final answers should not contain negative exponents.)

11. $2^{2/3} \cdot 2^{-5/3}$

12. $\frac{q^{5/4}}{q^{1/4}}$

13. $(x^{2/3} x^{1/2})^6$

14. $\left(\frac{m^{-1/4}}{n^{-1/2}}\right)^{-4}$

15. $(a^{-1/3} b^{1/2})^4 (a^{-1/2} b^{3/5})^{10}$

7.3 – Simplifying Radicals

*Evaluate or Simplify each:

1. $\sqrt{50}$

2. $\sqrt{108}$

3. $\sqrt[3]{54}$

4. $\sqrt{98a^5b^7}$

5. $\sqrt[5]{32x^{31}y^{87}}$

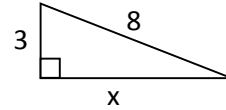
6. $\sqrt{\frac{25}{36}}$

7. $\sqrt[4]{(-4)^4}$

8. $\sqrt{60x^3}$

9. $\sqrt[3]{16x^2y^5}$

10. Find x:



11. Simplify: $\sqrt{a^2 + b^2}$

12. Simplify: $\sqrt{x^2 + 4x + 4}$

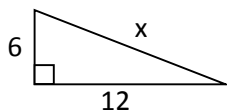
13. $\sqrt[3]{-192a^6bc^2}$

14. $\frac{7\sqrt{18}}{9}$

15. $\sqrt[3]{\frac{-27a^4}{8a}}$

16. $\sqrt{7^5u^{12}v^{20}w^{65}x^{80}}$

17. Find x:



7.4 – Adding & Subtracting Radicals

⇒ Property: $\sqrt[n]{a} + \sqrt[n]{a} = 2\sqrt[n]{a}$

*Combine. Simplify if possible.

1. $5\sqrt{7} + 9\sqrt{7}$

2. $8\sqrt[3]{2} + 7\sqrt{2} - 5\sqrt[3]{2}$

3. $3\sqrt{80} - 5\sqrt{45}$

4. $7\sqrt{72} - \sqrt{8} + 4\sqrt{50}$

5. $x^3\sqrt{64x^5y^2} - x^2\sqrt[3]{x^2y^2} + 5\sqrt[3]{64x^8y^2}$

7.5 – Multiplying Radicals

⇒ Property: $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$

*Multiply. Simplify, if possible.

1. $\sqrt{3} \cdot \sqrt{2}$

2. $5\sqrt{2}(7\sqrt{11})$

3. $(3\sqrt[3]{7})(5\sqrt[3]{2})$

4. $(2\sqrt[4]{3x})(4\sqrt[4]{27x^6})$

5. $(8\sqrt{7} - \sqrt{5})(\sqrt{7} + 3\sqrt{5})$

6. $(6 - \sqrt{11})^2$

7. $\sqrt{x} \cdot \sqrt[4]{x}$

8. $\sqrt[3]{5w} \cdot \sqrt[4]{5w}$

9. $\frac{\sqrt{u^3}}{\sqrt[3]{u}}$

10. $\sqrt[4]{8} \cdot \sqrt{3}$

7.6 – Dividing Radicals

Key idea: An expression is not simplified if its denominator contains a radical.

Vocabulary: The process of removing the radical from the denominator is called

*Simplify each of the following:

1. $\frac{\sqrt{7}}{\sqrt{16}}$

2. $\frac{\sqrt{21}}{\sqrt{3}}$

3. $\frac{4}{\sqrt{5}}$

4. $\frac{4}{\sqrt{6}}$

5. $\frac{5}{2+\sqrt{3}}$

6. $\frac{6}{3\sqrt{2}+4\sqrt{3}}$

7. $\frac{4\sqrt{3}+2\sqrt{5}}{-3\sqrt{3}-3\sqrt{5}}$

$$8. \frac{1}{\sqrt[11]{a^2 b^7 c^8}}$$

$$9. \frac{1}{\sqrt[3]{9b^2}}$$

$$10. \frac{2}{\sqrt[3]{7}}$$

$$11. \sqrt[4]{\frac{81}{6}}$$

$$12. \text{Rationalize: } \frac{\sqrt[4]{a^2 b^3}}{\sqrt[6]{ab^2}}$$

7.7 – Radical Equations

Process:

1. Isolate the radical.
2. Square (or cube) each side of the equation.
3. Solve the equation.
4. Check your solutions.

1. Solve: $\sqrt{9z - 5} - 2 = 9$

2. Solve: $(k + 18)^{1/3} + 2 = 0$

3. Solve: $\sqrt{b^2 - 5b - 8} = b + 7$

4. Solve: $\sqrt{5y + 1} + 2 = y + 3$

6. Solve: $\sqrt{p + 7} = \sqrt{2p} + 1$

5. Solve: $\sqrt{5a - 9} = \sqrt{5a} - 3$

7.8 – Complex and Imaginary Numbers

Definition #1: $\sqrt{-1} = \underline{\hspace{2cm}}$

Definition #2: $i^2 = \underline{\hspace{2cm}}$

1. Simplify: $\sqrt{-25}$

2. Simplify: $-\sqrt{-25}$

3. Add: $\sqrt{-16} + \sqrt{-36}$

4. Multiply: $\sqrt{-3}(\sqrt{-6})$

5. Complex numbers are of the form $\underline{\hspace{2cm}}$.

6. Add: $(-3 + 5i) + (7 - 2i)$

7. Subtract: $(-3 + 5i) - (7 - 2i)$

8. Multiply: $(-3 + 5i)(7 - 2i)$

9. Multiply $-4 + 7i$ by its conjugate.

10. Divide: $\frac{-3+5i}{7-2i}$

11. Divide: $\frac{-6+3i}{4-2i}$

Complex numbers allow us to factor the sum of two squares:

Factor: $x^2 + 25$

Factor: $x^2 + 18$

Powers of i:

$i =$

$i^2 =$

$i^3 =$

$i^4 =$

12. Simplify: i^{51}

13. Simplify: i^{101}

14. Simplify: i^{80}

Some Chapter 7 Review Problems

1. Simplify: $\sqrt[3]{-24x^5y^7z^9}$

2. Evaluate: $16^{-1/4}$

3. Write $\sqrt[4]{x^3}$ using rational exponents.

4. Add: $x\sqrt[3]{16x^2} - 4\sqrt[3]{2x^5} + 5x\sqrt[3]{54x^2}$

5. Simplify: $\sqrt{\frac{5}{3w}}$

6. Simplify: $\frac{-2}{\sqrt[3]{2x^2}}$

7. Divide: $\frac{-6}{\sqrt{7}-\sqrt{5}}$

8. Solve: $\sqrt{x+2} = 1 - \sqrt{2x+5}$