

Lecture Guide

Math 90 - Intermediate Algebra

to accompany

Intermediate Algebra, 3rd edition

Miller, O'Neill, & Hyde

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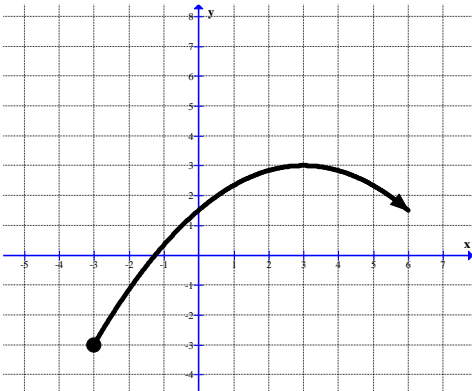
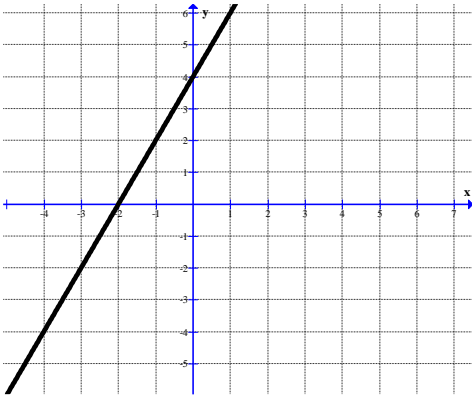
9.1 – Inverse Functions

A function is _____
if it passes both a vertical and horizontal line test. If a function is one-to-one, then it is _____ (it has an inverse which is also a function).

To find the inverse of a function from its **equation**, switch the x and y , and then solve for the "new" y .

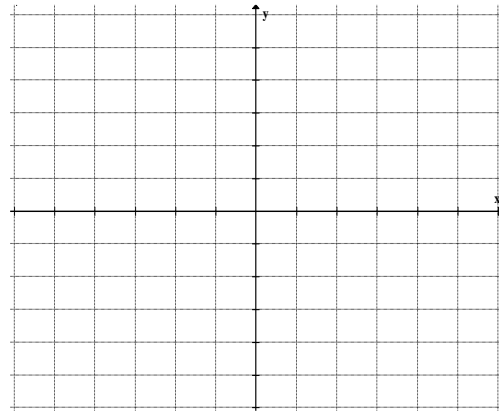
To find the inverse of a function from its **graph**, reflect the graph across the line _____.
If (a, b) is on $f(x)$, then (b, a) is on the graph of its inverse.

*Given the graph of $f(x)$, graph its inverse.

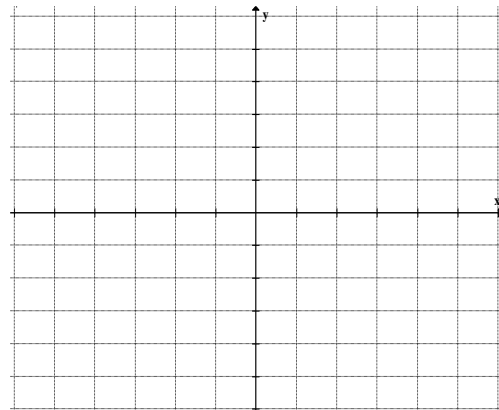


*Find the inverse of each function and then graph each on the same graph:

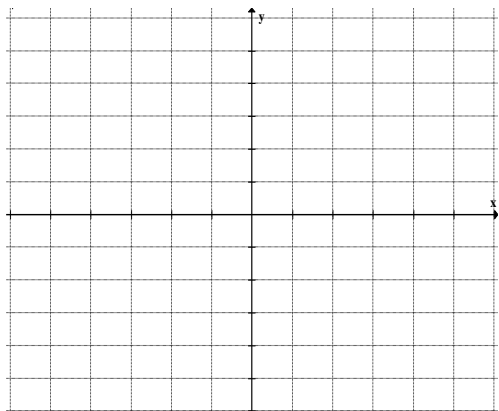
1. $g(x) = 3x - 2$



2. $h(x) = x^2 + 2$



3. $f(x) = \sqrt{x+1}$



To prove algebraically that two functions are inverses of each other, show that $f \circ f^{-1}(x) = x$ and $f^{-1} \circ f(x) = x$.

4. Prove that $f(x) = \sqrt[3]{x+5}$ and $g(x) = x^3 - 5$ are inverses of each other.

5. The one-to-one functions g and h are defined as follows:

$$g = \{(-7,1), (1,5), (2,0), (5,-4), (6,3)\}$$

$$h(x) = 2x + 3$$

Find the following:

$$g^{-1}(5) =$$

$$h^{-1}(x) =$$

$$(h^{-1} \circ h)(-4) =$$

6. Given $f(x) = \sqrt{-x-2}$ and $g(x) = |x| + 2$, find the following:

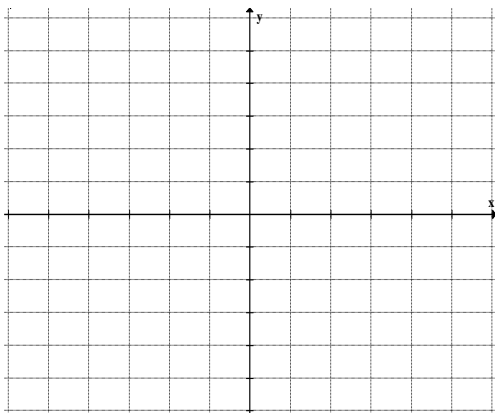
a. $g \circ f(x)$

b. the domain of $g \circ f(x)$.

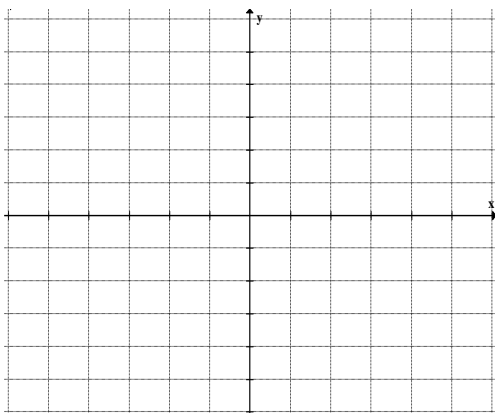
9.2, 9.3 – Exponential & Logarithmic Functions

$y = a^x$ is an exponential equation.

1. Graph: $y = 2^x$



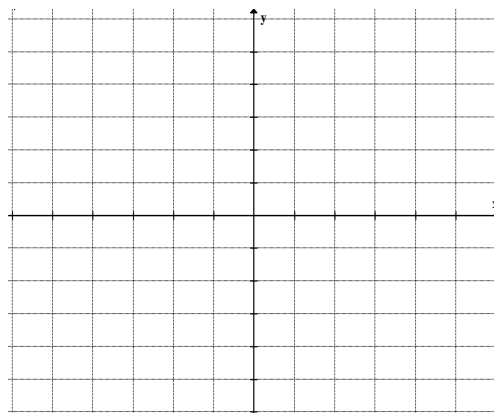
2. Graph: $y = \left(\frac{1}{2}\right)^x$



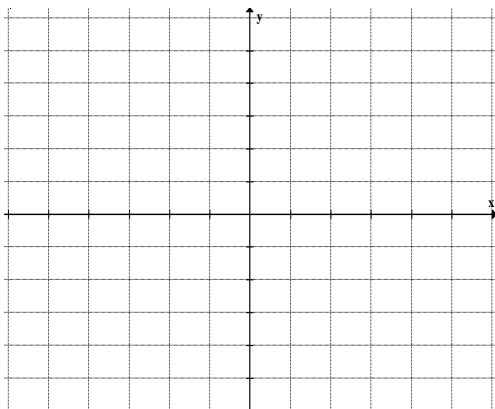
Convert each from logarithmic form to exponential form (or vice versa):

Logarithmic Form	Exponential Form

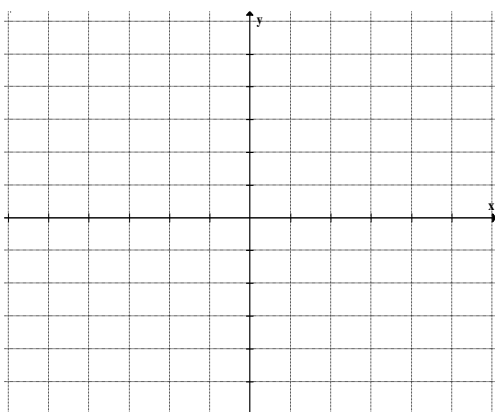
3. Graph $y = \log_2 x$



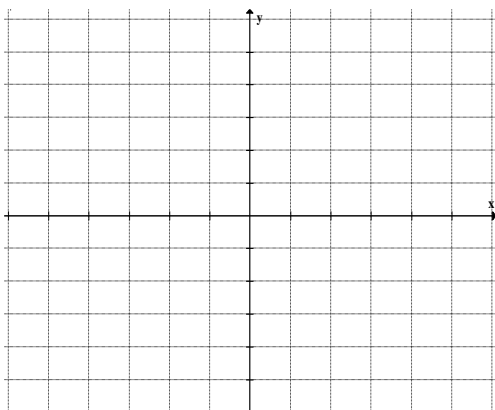
4. Graph: $f(x) = 4\log_2 x + 1$



5. Graph: $f(x) = \frac{1}{2}\log_3(x + 1) - 2$



6. Graph: $f(x) = 3\log_4(x - 2) + 1$



7. State the domain of $f(x) = \log_4(x + 7)$

8. SUMMARY OF DOMAINS: Find the domain of each function. Write each using interval notation.

a. $f(x) = \frac{x+1}{x-2}$

b. $g(x) = \sqrt{x-2}$

c. $h(x) = \log(x-2)$

Solve each equation:

9. $\log_x 16 = 4$

10. $\log_3 x = 5$

11. $\log_5 25 = x$

Evaluate each of the following:

12. $\log_3 81$

13. $\log_{10}(10,000)$

14. $\log_2 \left(\frac{1}{64} \right)$

15. If \$12,000 is borrowed for 15 years at 3.75% interest, compounded annually, and then paid in full at the end of that period, how much must be paid back at that time?

Use $A = P(1 + r)^t$

16. If \$15,000 is borrowed for 12 years at 4.5% interest, compounded annually, and then paid in full at the end of that period, how much must be paid back at that time?

9.4 – Properties of Logarithms

Properties:

1. $\log(xy) = \log x + \log y$

2. $\log\left(\frac{x}{y}\right) = \log x - \log y$

3. $\log(x^p) = p \cdot \log x$

4. $\log_a(1) = 0$

5. $\log_a(a) = 1$

6. $\log_a(a^x) = x$

7. $a^{\log_a x} = x$

*Write each as separate, simplified logarithms:

1. $\log(xy^2)$

2. $\log\left(\frac{\sqrt{x}}{yz}\right)$

3. Write as a single, simplified log:

$$\log_5 a - \frac{1}{2} \log_5 b - 3 \log_5 c$$

4. Write as a single, simplified log:

$$\log_x(p^2 - 4) - \log_x(p - 2)$$

5. Fill in the missing values that make the equations below true:

(a) $\log_4 3 + \log_4 7 = \log_4 \square$

(b) $\log_5 8 - \log_5 \square = \log_5 \frac{8}{9}$

(c) $2 \log_8 5 = \log_8 \square$.

9.5 – Common and Natural Logarithms

Calculators: Exponents and Logarithms

Exponential keys:

Logarithmic keys:

Definition: As $x \rightarrow +\infty$, $\left(1 + \frac{1}{x}\right)^x \rightarrow e$

On your calculator, find the following values:

1. $(3.07)^{1.42} \approx$

2. $e \approx$

3. $\log(4387) \approx$

4. $\ln(317) \approx$

5. $e^{3.78} \approx$

6. $\log_7(56) \approx$

Change-of-Base Formula: $\log_b n = \frac{\log n}{\log b}$

7. $\log_5(45) \approx$

Compound interest: $A = P \left(1 + \frac{r}{n}\right)^{nt}$

8. \$1000 is invested at 1.2% for 10 years. Find the value of the account after 10 years if the interest is compounded....

a. quarterly.

b. monthly.

c. daily.

d. continuously. Use $A = Pe^{rt}$.

9.6 – Exponential Equations

A. Exponential – Same Base

1. Solve: $2^{-x} = 64$

2. Solve: $4^{2x-7} = \frac{1}{128}$

3. Solve: $16^{-x+1} = 8^{5x}$

B. Exponential – Different Base

4. $8^x = 21$

5. $3^{x+1} = 5^x$

6. $e^x = 0.3151$

7. If \$1500 is invested at 2.25%, compounded monthly, how long would it take to grow into \$1800?

8. For how long should \$1000 be invested at 1.1% compounded daily in order for the money to double?

9.7 – Logarithmic Equations

1. Solve: $\log_5(c^2 - c) = \log_5 20$

2. Solve: $\ln(y - 4) = \ln(2y - 1)$

3. Solve: $\log_3 8 - \log_3(x + 5) = 2$

4. Solve: $\log_2(h - 1) + \log_2(h + 1) = 3$

5. Solve: $\log_3 k + \log_3(2k + 3) = 2$

6. Solve: $\log_4(6y - 7) + \log_4 3 = \log_4 5$

10.1 – Distance and Midpoint

Distance Formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Midpoint Formula: $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

Derrivation of Formulas:

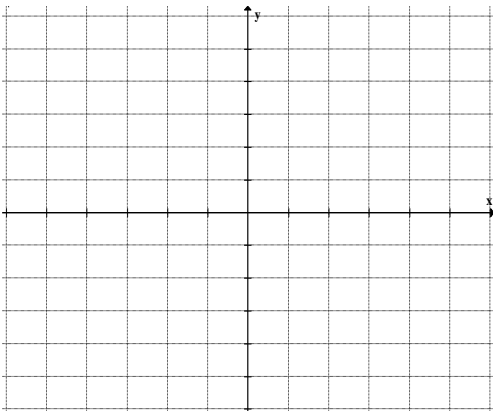
1. Find the distance and midpoint between $(-3,5)$ and $(7,2)$.

2. Find the distance and midpoint between $\left(4, \frac{5}{2}\right)$ and $(-5, -1)$.

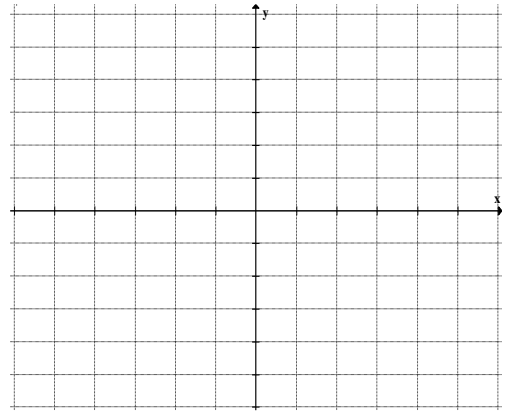
Chapter 9 Review

1. Solve: $-\log_2(x - 4) = 3 - \log_2(x + 3)$

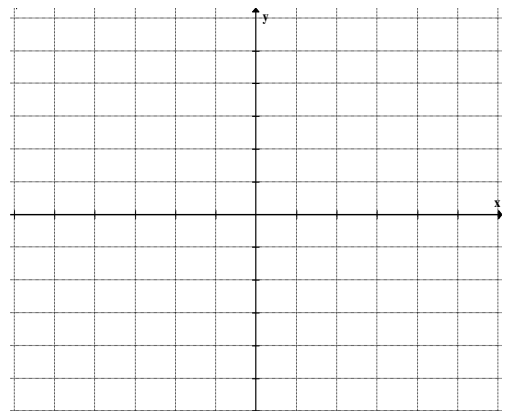
2. Graph: $g(x) = 3\log_4(x - 2) + 1$



3. Graph: $h(x) = -2\log_2(x + 1) + 3$



4. Graph $y = 3^{x+2} - 1$



5. Solve: $3^{x+4} = 81$

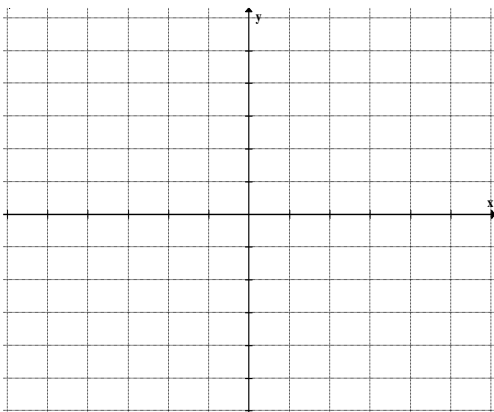
6. Solve: $3^{x+4} = 85$

7. Solve: $\log x - 1 = -\log(x - 9)$

8. Solve: $\ln(21) = 1 + \ln(x - 2)$

9. For how long should \$800 be invested at 4.3%, compounded daily, in order for it to grow to \$2000?

10. Find and graph the inverse of $f(x) = 3x^2 - 1$



11. Write as a separate, simplified logarithm:

$$\log \sqrt[6]{\frac{xy^2}{z}}$$

12. Evaluate $\log_3 \left(\frac{1}{27} \right)$

13. Evaluate $\log_5 75$

14. Solve: $e^{3x} = 17$

15. Given $f(x) = \sqrt{x+3}$ and $g(x) = x^2 - 1$, find the following:

a. $(f + g)(1)$

b. $f \circ g(x)$

c. $g \circ f(4)$

d. the domain of $f + g$, $f - g$, and fg .

e. the domain of $\frac{f}{g}$.

16. Given $f(x) = \frac{2x+3}{7x-8}$, find the following:

a. $f^{-1}(x)$

b. the domain of $f^{-1}(x)$.

c. the range of $f^{-1}(x)$.