5.1-5.2 <u>Classical Probability</u>

In classical probability, we assume that all

outcomes are____________________.

example: **flipping a coin...**

P(heads)=_________P(tails)=__________

example: **rolling a die...**

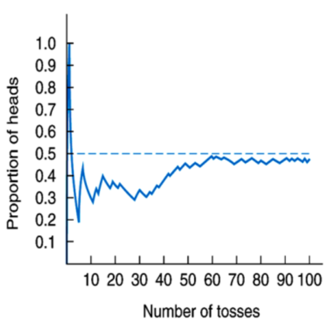
P(4)=_________P(odd)______P(7)=______

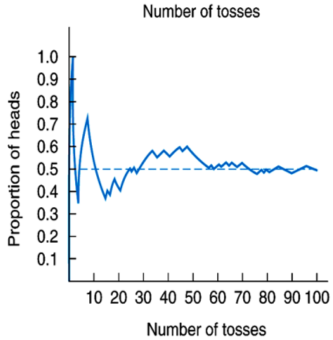
BASIC PROPERTIES:

1. P(E) is always between___ and____.
2. The probability of an impossible event is__.
3. The probability of a certain event is_____.

The **<u>frequential interpretation of probability</u>** construes the proportion of times it occurs in a large number of repetitions of the event.

Two computer simulations of tossing a balanced coin 100 times:





<u>Dice Chart:</u>

	1	2	3	4	5	6
1						
2						
3						
4						
5						
6						

Find the following probabilities:

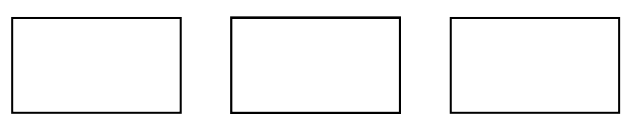
P(2)=_______ P(7)=_______

P(multiple of 5)=__________

Sample Space -

For any event E, there is a corresponding event defined by the condition "E does not occur." It is called the **<u>complement of E</u>** and is denoted by "**not E**."

Venn Diagrams:



not E A & B A or B

<u>Definitions:</u> Suppose A and B are events.

not A: the event that "A does not occur"

A&B: the event that **<u>both</u>** event A and event B occur

A or B: the event that **<u>either</u>** event A **<u>or</u>** event B occur

Example: A={1,2,3} B={1,3,5} C={4,5,6)

$A \cup B$ ____________ $A \cup C$ ____________

$A \cap B$ ____________ $A \cap C$ ____________

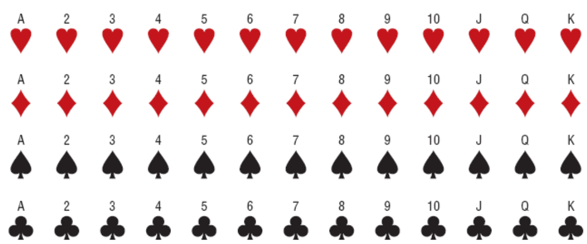
example: A die is tossed. Consider the following events:

A= the event that an even is rolled
B= the event that an odd is rolled
C= the event that a 1, 2, or 3 is rolled.

List the outcomes which comprise each event:

A&B________**A&C**_______**not C**_______.

A or B____________**A or C**____________



example: Consider a shuffled deck of 52 cards and the following events:

A= the event that a club is chosen
B= the event that a face card is chosen
C= the event that the 6 of spades is chosen
D= the event that a 6 is chosen

Find the following probabilities:

P(A)=_____________**P(B)=**_____________

P(C)=_____________**P(D)=**_____________

Describe the following in words:

not A:

A & D:

A or C:

The **<u>odds</u>** that an event occurs can be found using the ratio of the number of ways it *can* occur to the number of ways it *cannot* occur:

Example: Find the odds of rolling a two with a single die.

Example: A class contains 18 men and 14 women.

a) Find the probability of choosing a woman at random.

b) Find the odds of choosing a woman at random.

A **tree diagram** is a device used to determine all possible outcomes of a probability experiment.

Use a tree diagram to find the sample space for the gender of three children in a family.

The Addition Rules for Probability

Two events are said to be________________ if they cannot both occur at the same time.

A collection of ________________________

and____________________________events occur if:

a) each event is mutually exclusive of all others; and
b) the union of the events is the sample space.

Example:

Addition Rule: P(A or B) = P(A) + P(B) when events A and B are mutually exclusive.

General Addition Rule:
P(A or B) = P(A) + P(B) - P(A & B) when A and B are not necessarily mutually exclusive.

Complement Rule: P(E) = 1 - P(not E)

example: Roll a die...

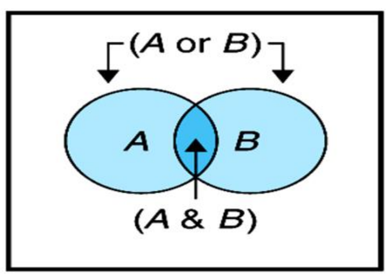
A = event that a 3 is rolled
B = event that a 2 is rolled
C = event that a number less than 3 is rolled

P(A)=_______ P(A or B)=_________

P(B)=_______ P(not A)=__________

P(C)=_______ P(B or C)=_________

Example: A card is chosen at random from a deck of 52 cards. Find the probability of choosing a heart or a queen.



5.3 Multiplication Rules and Conditional Probability

Contingency tables give a frequency distribution for cross-classified data. The boxes inside are each called **cells**.

example: The following contingency table provides a cross-classification of U.S. hospitals by type and number of beds:

		24-beds	25-74 beds	75+ beds	
TYPE		B1	B2	B3	
General	H1	260	1586	3557	5403
Psychiatric	H2	24	242	471	737
Chronic	H3	1	3	22	26
Tuberculosis	H4	0	2	2	4
Other	H5	25	177	208	410
		310	2010	4260	6580

a) Describe each of the following **in words**:

H2

B2

(H2 & B2)

(H2 or B2)

b) Compute the probability of each above.

P(H2)=________________________________

P(B2)=__________________________

P(H2&B2)=______________________

P(H2 or B2)=____________________

d) Construct a joint probability distribution:

		24-beds	25-74 beds	75+ beds	
TYPE		B1	B2	B3	
General	H1				
Psychiatric	H2				
Chronic	H3				
Tuberculosis	H4				
Other	H5				
					1.000

The conditional probability of an event A, given that B occurs, is given by $P(A|B)=\frac{P(A\&B)}{P(B)}$.

example: Roll a die...

A= the event that a 3 is rolled
B= the event that an odd is rolled

P(A)=_______

P(B)=_______

P(A&B)=__________

P(A or B)=_________

P(A|B)=___________

P(B|A)=___________

example: The table below provides a joint probability distribution for the members of the 105^{th} Congress by legislative group and political party.

		House	Senate	
		C1	**C2**	
Democrats	**P1**	**0.385**	**0.084**	**0.469**
Republicans	**P2**	**0.424**	**0.103**	**0.527**
Other	**P3**	**0.004**	**0.000**	**0.004**
		0.813	**0.187**	**1.000**

If a member of the 105^{th} Congress is selected at random, what is the probability that the member obtained

a) is a senator?

b) is a Republican senator?

c) is a Republican, given that he or she is a senator?

d) is a senator, given that he or she is a Republican?

Class Example:

	Male	Female	Total
Chocolate			
Strawberry			
Vanilla			
Total			

<u>Multiplication Rule</u>: P(A&B)= P(A)*P(B|A)

example: In Mr. Toner's math class, the male/female ratio is 17:23. Select 2 students at random. Assume that the first student chosen is not allowed to be chosen a second time. Find the probability of selecting a girl first, then a guy second.

Draw and label a tree diagram for the experiment.

Example: A bag contains 3 red and 4 white marbles. Choose 2 marbles out, one at a time. Draw a tree diagram for this problem both with replacement and without replacement.

with replacement:

without replacement:

What is the difference between independent and dependent trials?

Example: Box 1 contains 2 red balls and 1 blue ball. Box 2 contains 3 blue balls and 1 red ball. A coin is tossed. If it lands heads up, box 1 is selected and a ball is drawn. If it lands tails up, box 2 is selected and a ball is drawn. Find the probability of selecting a red ball.

Example: A coin is flipped six times. Find the probability that at least one of the flips will contain a tails.

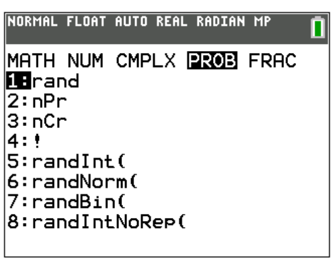
5.4 <u>Counting Rules</u>

Fundamental Counting Rule- When 2 events are to take place in a definite order, with m_1 possibilities for the first event and m_2 possibilities for the second event, then there are $m_1 \cdot m_2$ possibilities altogether. In general, for **k** events, multiply $m_1 \cdot m_2 \cdots m_k$

example: license plate

Factorial notation:

You can find the factorial, permutation, and combination keys on your TI-84 in the **MATH** menu in the **PROB** column.



<u>Permutation</u>- a collection or arrangement of

objects in which___________is important.

The number of permutations of **r** objects from a group of **n** objects is given by the formula

$${}_nP_r = \frac{n!}{(n-r)!}.$$

examples:

1. In a race of 7 runners, in how many ways can they place 1st, 2nd, and 3rs?

2. A club has 25 students in it. In how many ways can they choose a president and vice president?

3. In how many ways can 5 kids stand in a line at a drinking fountain?

<u>Combination</u>- a collection of objects in which order is **<u>not</u>** important.

The number of combinations of r objects from a group of n objects is given by the formula:

$${}_nC_r = \frac{n!}{r!\cdot(n-r)!}$$

examples:

1. In how many ways can Mr. Toner choose 2 of his 40 students to give free passes to Disneyland?

2. In how many ways can 5 cards be chosen from a shuffled deck?

3. In how many ways can you choose 5 numbers out of 47 and one MEGA number out of 27 numbers in the state lottery?

Example

A newspaper editor has received 8 books to review. He decides that he can use 3 reviews in his newspaper. How many different ways can these 3 reviews be selected?

$$_8C_3 = \frac{8!}{(8-3)!3!} = \frac{8!}{5!3!} = \frac{8\cdot 7\cdot 6}{3\cdot 2\cdot 1} = 56 \text{ ways}$$

Example

In a club there are 7 women and 5 men. A committee of 3 women and 2 men is to be chosen. How many different possibilities are there?

Solution: You must select 3 women from the 7 women, which can be done in $_7C_3 = 35$ ways. Then 2 men must be selected from the 5 men, which can be done in $_5C_2 = 10$ ways. Finally, by the fundamental counting rule, the total number of possibilities can be found by multiplying.

$$_7C_3 \cdot {}_5C_2 = \frac{7!}{(7-3)!3!} \cdot \frac{5!}{(5-2)!2!} = 350 \text{ ways}$$

Selected extra exercises:

a. The call letters of a radio station must have 4 letters. The first letter must be a K or a W. How many different station call letters can be made if repetitions are not allowed? How many different station call letters can be made if repetitions are allowed?

b. How many different ways can a city health department inspector visit 5 restaurants in a city with 10 restaurants?

c. How many different 4-letter permutations can be formed from the letters in the word ***decagon***?

d. How many different 4-letter permutations can be formed from the letters in the word Mississippi?

e. A particular cell phone company offers 4 models of phones, each in 6 different colors and each available with any one of 5 calling plans. How many combinations are possible?

f. If a person can select 3 presents from 10 presents under a Christmas tree, how many different combinations are there?

g. In a train yard, there are 4 tank cars, 12 boxcars, and 7 flatcars. How many ways can a train be made up consisting of 2 tank cars, 5 boxcars, and 3 flatcars? (In this case, order is not important.)

h. There are 16 seniors and 15 juniors in a particular social organization. In how many ways can 4 seniors and 2 juniors be chosen to participate in a charity event?

Example

A store has 6 tabloid magazines and 8 news magazines on the counter. If two customers purchased a magazine, find the probability that one of each magazine was purchased.

P (1 tabloid and 1 news) =

$$= \frac{{}_6C_1 \cdot {}_8C_1}{{}_{14}C_2} = \frac{6 \cdot 8}{91} = \frac{48}{91}$$

i. Find the probability of selecting 3 science books and 4 math books from 8 science books and 9 math books. The books are selected at random.

j. In problem **h** (left), what is the probability that 4 seniors and 2 juniors are the six chosen to participate in a charity event?

6.1 Probability Distributions

A discrete random variable is a random variable whose possible values form a discrete data set, only taking on certain values.

example:

# of rooms in a home	x	1	2	3	4	5
	P(x)	0.054	0.173	0.473	0.281	0.020

Find P(x=3)=_______.

In the next example you are given frequencies, rather than probabilities:

example: The following table displays a frequency distribution for the enrollment by grade in public secondary schools. Frequencies are in thousands of students.

Grade	9	10	11	12
Frequency	3604	3131	2749	2488

Suppose a student in secondary school is to be selected at random. Let x denote the grade level of the student chosen. Determine P(x=10) and interpret your results in terms of percentages.

Two Requirements for a Probability Distribution:

1. The sum of the probabilities of all events must equal 1.
2. The probability of each event in the sample space must be between o and 1. That is, $0 \le P(E) \le 1$.

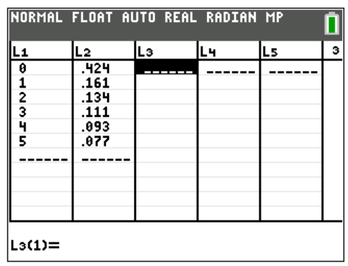
Mean, Variance and Expectation

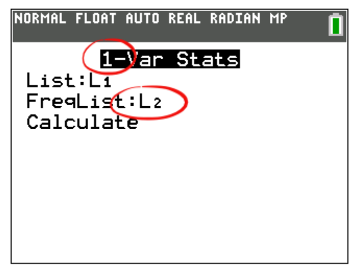
The mean of a probability distribution is given the special name expected value, defined by $\mu_x = (\Sigma(x \cdot p(x)))$. This means that for a large number of observations of the random variable **x**, the mean (or expected value) will be approximately μ_x.

example: The following is a probability distribution for the number of customers waiting at Benny's Barber Shop in Cleveland:

x	p(x)
0	0.424
1	0.161
2	0.134
3	0.111
4	0.093
5	0.077

TI-84 Procedure:





Interpretation: If we were to enter the barber shop a large number of times, we would expect approximately 1.519 people to be waiting in line. Could this happen? Explain.

What is the meaning of the standard deviation in this context? It measures the dispersion of the possible values of x relative to the mean. In the example above, we'd expect 1.519 people waiting in line at the barber shop with a standard deviation of 1.674 people.

example: Suppose a lottery contest allowed you to spin a wheel for a prize. On the wheel, each outcome is equally-likely. Find the expected winnings and standard deviation if the prizes are distributed as follows...

Prize x	p(x)
$250	**0.01**
$175	**0.04**
$150	**0.08**
$100	**0.12**
$75	**0.25**
$50	**0.50**

Answers and Interpretation:

One thousand tickets are sold at $1 each for a color television valued at $350. What is the expected value of the gain if a person purchases one ticket?

A landscape contractor bids on jobs where he can make $3000 profit. The probabilities of getting 1, 2, 3, or 4 jobs per month are shown.

Number of jobs	1	2	3	4
Probability	0.2	0.3	0.4	0.1

Find the contractor's expected profit per month.

6.2 The Binomial Distribution

A **binomial experiment** is a probability experiment that satisfies the following three requirements:

1.

2.

3.

Examples of binomial trials:

A population in which each member is classified as either having or not having a

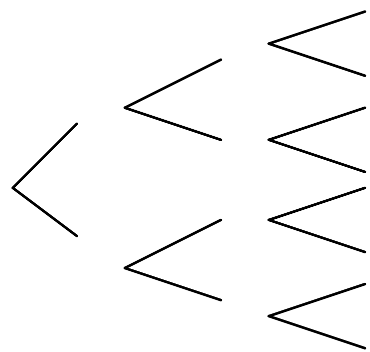
specific attribute is called a ______________ ____________population.

Suppose a survey were done of all U.S. households to see if they own a microwave. The population to be surveyed would be huge! We cannot get exact percentages, but only an estimation.

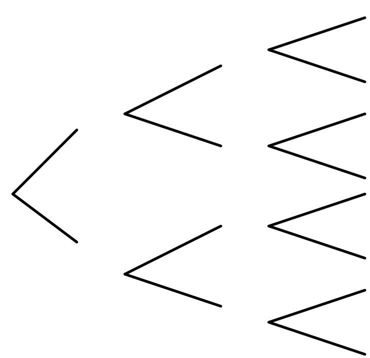
When running this survey, the sampling could ***either*** be done **with** or **without** replacement. Suppose you had a huge list containing every person's name in the U.S. If you were to cross off names as you surveyed people, so that you would not call them twice, then you would be surveying **without** replacement.
Would it make a difference if you crossed out names if you had a huge list of names and you were doing random sample surveying? Explain.

Rule of thumb: If a sample size is less than 5% of a population size, then Bernoulli (independent) trials may be assumed (and surveying can be done with replacement).

example: Draw a tree diagram for flipping a coin three times.



example: Draw and label a tree diagram for flipping a coin three times if the coin is bent and has a 75% chance of landing on "heads" each time it is flipped. Find and label the sample space and each of the associated probabilities.



Suppose n binomial trials are to be performed. The probability distribution for ***x*** successes in ***n*** binomial trials is given by

$$P(x) = \binom{n}{x} p^x (1-p)^{n-x},$$

where n= # of trials, x= # of successes, p= probability of a success

On the TI-84, we use the binompdf and binonmcdf functions, found in the **DIST** menu:

Binom**p**df(numtrials, probsuccess, numsuccesses) finds $P(x = \#)$
Binom**c**df(numtrials, probsuccess, numsuccesses) finds $P(x \leq \#)$

example: A salesperson makes 8 contacts per day with potential customers. From past experience, we know that the probability a potential customer will purchase a product is 0.10.

a) What is the probability that he/she makes exactly 2 sales on a particular day?

b) What is the probability he/she makes at most 2 sales on a particular day?

c) What is the probability he/she makes at least 2 sales on a particular day?

PATTERNS:

examples:

1. A true/false test has 15 questions on it. If you randomly guess at each question, what is…

a) P(x=6 correct)______________________

b) P(x>11 correct)______________________

c) P(x<9 correct)______________________

2. A 10 question multiple choice test has 5 possible responses for each question. If you randomly guess at each question, what is

a) P(x=8 correct)______________________

b) P(x$\geq$6 correct)______________________

b) P(x$\leq$7 correct)______________________ example: According to the US Census Bureau, 25% of US children are not living with both parents. If 10 US children are selected at random, determine the probability that the number not living with both parents is...

a) exactly two.___________

b) at most two.___________

c) between three and six, inclusive.________

Binomial Expected Values

example: As reported by Television Bureau of Advertising, Inc., in *Trends in Television,* 84.2% of U.S. households have a VCR. If six households are randomly selected without replacement, what is the (approximate) probability that the number of households sampled that have a VCR will be

1. exactly four?

2. at least four?

3. At most five?

4. Between two and five, inclusive?

5. Determine the (approximate) probability distribution of the random variable Y, the number of households of the six sampled that have a VCR.

6. Determine and interpret the mean of the random variable Y.

7. Obtain the standard deviation of Y.

When flipping a coin 6 times in a row, find the probability of getting at least 5 heads.

6.3 <u>The Poisson Distribution</u>

A type of probability distribution that is often useful in describing the number of events that will occur in a specific amount of time or in a specific area or volume is the **Poisson distribution**. Typical examples of random variables for which the Poisson probability distribution provides a good model are:

1. The number of traffic accidents per month in a busy intersection.
2. The number of noticeable surface defects (scratches, dents, etc.) found by quality inspectors on a new automobile.
3. The parts per million of some toxin found in the water or air emission from a manufacturing plant.
4. The number of diseased trees per acre of a certain woodland.
5. The number of death claims received per day by an insurance company.
6. The number of unscheduled admissions per day to a hospital.

Characteristics of a Poisson Random Variable

1. The experiment consists of counting the number of times a certain event occurs during a given unit of time or in a given area or volume (or weight, distance, or any other unit of measure).
2. The probability that an event occurs in a given unit of time, area, or volume is the same for all the units.
3. The number of events that occur in one unit of time, area, or volume is independent of the number that occur in other units.
4. The mean (or expected) number of events in each unit is denoted by the Greek letter, lambda, λ, and the standard deviation is $\sqrt{\lambda}$.

The characteristics of the Poisson random variable are usually difficult to verify for practical examples. The examples given satisfy them well enough that the Poisson distribution provides a good model in many instances. As with all probability models, the real test of the adequacy of the Poisson model is in whether it provides a reasonable approximation to reality- that is, whether empirical data support it.

The Poisson Distribution is used to model the frequency with which an event occurs during a particular period of **time** using $p(x) = e^{-\lambda}\left(\frac{\lambda^x}{x!}\right)$, where λ (lambda) is given and $e \approx 2.71828$. The expected value of a Poisson distribution is given by $\mu_x = \lambda$, with $\sigma_x = \sqrt{\lambda}$.

On the TI-84 **DIST** menu you can find poissonpdf and poissoncdf.

example: The owner of a fast food restaurant knows that, on the average, 2.4 cars (customers) use the drive-through window between 3:00 pm and 3:15 pm. Assuming that the number of such cars has a Poisson distribution, find the probability that, between 3:00 pm and 3:15 pm,

a) exactly two cars will use the drive-through window.

b) at least three cars will use the drive-through window.

Probability Review Problems (Ch 5-6)

1. On a quiz consisting of 3 true/false questions, an unprepared student must guess at each one. The guesses will be random.

A. List the different possible solutions.

B. What is the probability of answering all 3 questions correctly?

C. What is the probability of guessing incorrectly for all questions?

D. What is the probability of passing the quiz by guessing correctly for at least 2 questions?

2.

A. If a person is randomly selected, find the probability that his or her birthday is October 18, which is National Statistics Day in Japan. Ignore leap years.

B. If a person is randomly selected, find the probability that his or her birthday is in November. Ignore leap years.

3. After collecting IQ scores from hundreds of subjects, a boxplot is constructed with this 5-number summary: {82, 91, 100 , 109, 118}. If one of the subjects is randomly selected, find the probability that his or her IQ score is greater than 109.

4. Find the probability of getting 4 consecutive aces when 4 cards are drawn without replacement from a shuffled deck.

5. A typical "combination" lock is opened with the correct sequence of 3 numbers between 0 and 49 inclusive. How many different sequences are possible? (A number can be used mare than once.) Are these sequences combinations or are they actually permutations?

6. Mars, Inc., claims that 20% of its plain M&M candies are red. Find the probability that when 15 plain M&M candies are randomly selected, exactly 20% (or 3 candies) are red.

7. Suppose that a city has two hospitals. Hospital A has about 100 births per day, while Hospital B has only about 20 births per day. Assume that each birth is equally likely to be a boy or a girl. Suppose that for one year you count the number of days on which the a hospital has 60% or more of that day's births turn out to be boys. Which hospital would you expect to have more such days? Explain your reasoning.

8. a. If $P(A \text{ or } B) = \frac{1}{3}$, $P(B) = \frac{1}{4}$, and $P(A \text{ and } B) = \frac{1}{5}$, find $P(A)$.

b. If $P(A) = 0.4$ and $P(B) = 0.5$, what is known about $P(A \text{ or } B)$ if A and B are mutually exclusive events?

9. Ten percent of us are left-handed. What is the probability of randomly selecting two people who are both left-handed?

10. With one method of acceptance sampling, a sample of items is randomly selected without replacement and the entire batch is rejected if there is at least one defect. The Niko Electronics Company has just manufactured 5000 CD's, and 3% are defective. If 10 of the CD's are selected and tested, what is the probability that the entire batch will be rejected?

11. A typical "combination" lock is opened with the correct sequence of 3 numbers between 0 and 49 inclusive. How many different sequences are possible? (A number can be used more than once.) Are these sequences combinations or are they actually permutations?

12. In an age-discrimination case against Darmin Inc., evidence showed that among the last 40 applicants for employment, only the 8 youngest were hired. Find the probability of randomly selecting 8 of 40 people and getting the 8 youngest. Based on that result, does it appear that age-discrimination is occurring?

13. If you randomly select a person from the population of people who have died in recent years, there is a 0.0478 probability that the person's death was caused by an accident, according to data from the *Statistical Abstract of the United States*. A Baltimore detective is suspicious about 5 persons whose deaths were categorized as accidental. Find the probability that when 5 dead persons are randomly selected, their deaths were all accidental.

14. When you give a casino $5 for a bet on the number 7 in roulette, you have a 1/38 probability of winning $175 and a 37/38 probability of losing $5. What is your expected value? In the long run, how much do you lose for each dollar?

15. When you give a casino $5 for a bet on the "pass line" in the same game of craps, there is a 244/495 probability that you will win $5 and a 251/495 probability that you will lose $5. What is your expected value? In the long run, how much do you lose for each dollar bet?

16. According to the U.S. Department of Justice, 5% of all U.S households experienced at least one burglary last year, but Newport police report that a community of 15 homes experience 4 burglaries last year. By finding the probability of getting 4 or more burglaries in a community of at least 15 homes, does it seem that this community is just unlucky?

17. The following excerpt is from *The Man Who Cast Two Shadows*, by Carol O'Connell: "The child had only the numbers written on her palm in ink…, all but the last four numbers disappeared in a wet smudge of blood… She would put the coins into the public telephones and dial three untried numbers and then the four she knew. If a woman answered she would say, 'It's Kathy. I'm lost.' "If it costs Kathy 25 cents for each call and she tries every possibility except those beginning with 0 or 1, what is her total cost?